11. [15 points] The velociraptor population on the earth one year and four years after a huge meteor hits the earth is 2 million and 1.6 million respectively. Let $P$ be the velociraptor population (in millions) on the earth $t$ years after the meteor hits the earth.

a. [5 points] Suppose that the velociraptor population on the earth decreased exponentially after the meteor hits the earth. In this case, $P = g(t)$ for some function $g$. Find a formula for $g(t)$. Your answer should be in exact form.

Solution:

$$dg(t) = ab^t$$
$$2 = ab$$
$$1.6 = ab^4$$

$$\frac{ab^4}{ab} = 1.6 = 0.8$$
$$b^3 = 0.8 \quad b = (0.8)^{\frac{1}{3}} \quad a = \frac{2}{b} = \frac{2}{(0.8)^{\frac{1}{3}}}$$

$$g(t) = \frac{2}{(0.8)^{\frac{1}{3}}} \left((0.8)^{\frac{1}{3}}\right)^t$$

b. [4 points] Suppose that the velociraptor population on the earth is a power function of $t$, the number of years after the meteor hits the earth. In this case, $P = h(t)$ for some function $h$. Find a formula for $h(t)$. Your answer should be in exact form.

Solution:

$$h(t) = kt^p$$
$$2 = k(1)^p = k$$
$$1.6 = k(4)^p$$

$$1.6 = 24^p$$
$$4^p = 0.8$$
$$\ln(4^p) = \ln(0.8)$$
$$p \ln(4) = \ln(0.8)$$

$$p = \frac{\ln(0.8)}{\ln(4)} \quad h(t) = 2t^{\frac{\ln(0.8)}{\ln(4)}}$$

The problem continues on the next page
The velociraptor population on the earth one year and four years after a huge meteor hits the earth is 2 million and 1.6 million respectively. Let $P$ be the velociraptor population (in millions) on the earth $t$ years after the meteor hits the earth.

c. [1 point] Under which assumption does $P$ decrease faster to 0, if we assume that $P = g(t)$ or if we assume that $P = h(t)$? Circle your answer.

\[
\begin{align*}
\text{Solution:} & \quad P = g(t) \quad P = h(t) \quad \text{Cannot be determined.}
\end{align*}
\]

d. [3 points] Suppose that the velociraptor population on the earth decreased linearly after the meteor hits the earth. In this case, $P = f(t)$ for some function $f$. Find a formula for $f(t)$.

\[
\begin{align*}
\text{Solution:} & \quad m = \frac{2 - 1.6}{1 - 4} = -\frac{0.4}{3}, \text{ then } f(t) = 2 - \frac{0.4}{3}(t - 1).
\end{align*}
\]

e. [2 points] Give a practical interpretation of the horizontal intercept of the graph $P = f(t)$.

\[
\begin{align*}
\text{Solution:} & \quad \text{The number of years after the meteor hit earth needed to eradicate the population of velociraptors.}
\end{align*}
\]

12. [6 points] Let $N(x)$ be the cost (in dollars) to produce $x$ pieces of chocolate. The chocolates are then put into boxes containing ten pieces of chocolate each. The packaging costs for each box of chocolates is $0.15. Write down a mathematical expression describing the following.

a. [2 points] The average cost (in dollars per piece of chocolate) of producing $c$ chocolates.

\[
\begin{align*}
\text{Solution:} & \quad \frac{N(c)}{c}
\end{align*}
\]

b. [2 points] The cost in dollars of producing the fifteenth piece of chocolate.

\[
\begin{align*}
\text{Solution:} & \quad N(15) - N(14)
\end{align*}
\]

c. [2 points] The total cost in dollars (including packaging costs) of producing $b$ boxes of chocolate.

\[
\begin{align*}
\text{Solution:} & \quad 0.15b + N(10b).
\end{align*}
\]