11. [15 points] The velociraptor population on the earth one year and four years after a huge meteor hits the earth is 2 million and 1.6 million respectively. Let P be the velociraptor population (**in millions**) on the earth t years after the meteor hits the earth.

a. [5 points] Suppose that the velociraptor population on the earth decreased exponentially after the meteor hits the earth. In this case, $P = g(t)$ for some function g. Find a formula for $g(t)$. Your answer should be in **exact form**.

Solution:

$$\begin{aligned} dg(t) &= ab^t \\ 2 &= ab \\ 1.6 &= ab^4 \end{aligned}$$

$$\frac{ab^4}{ab} = \frac{1.6}{2} = 0.8$$

$$b^3 = 0.8 \qquad b = (0.8)^{\frac{1}{3}} \qquad a = \frac{2}{b} = \frac{2}{(0.8)^{\frac{1}{3}}}$$

$$g(t) = \frac{2}{(0.8)^{\frac{1}{3}}}\left((0.8)^{\frac{1}{3}}\right)^t$$

b. [4 points] Suppose that the velociraptor population on the earth is a power function of t, the number of years after the meteor hits the earth. In this case, $P = h(t)$ for some function h. Find a formula for $h(t)$. Your answer should be in **exact form**.

Solution:

$$\begin{aligned} h(t) &= kt^p \\ 2 &= k(1)^p = k \\ 1.6 &= k(4)^p \end{aligned}$$

$$\begin{aligned} 1.6 &= 24^p \\ 4^p &= 0.8 \\ \ln(4^p) &= \ln(0.8) \\ p\ln(4) &= \ln(0.8) \\ p &= \frac{\ln(0.8)}{\ln(4)} \end{aligned} \qquad h(t) = 2t^{\frac{\ln(0.8)}{\ln(4)}}$$

The problem continues on the next page

The velociraptor population on the earth one year and four years after a huge meteor hits the earth is 2 million and 1.6 million respectively. Let P be the velociraptor population (**in millions**) on the earth t years after the meteor hits the earth.

c. [1 point] Under which assumption does P decrease faster to 0, if we assume that $P = g(t)$ or if we assume that $P = h(t)$? Circle your answer.

Solution:

$\boxed{P = g(t)}$ $\quad P = h(t)$ $\quad$ Cannot be determined.

d. [3 points] Suppose that the velociraptor population on the earth decreased linearly after the meteor hits the earth. In this case, $P = f(t)$ for some function f. Find a formula for $f(t)$.

Solution: $m = \dfrac{2 - 1.6}{1 - 4} = -\dfrac{0.4}{3}$, then $f(t) = 2 - \dfrac{0.4}{3}(t - 1)$.

e. [2 points] Give a practical interpretation of the horizontal intercept of the graph $P = f(t)$.

Solution: The number of years after the meteor hit earth needed to eradicate the population of velociraptors.

12. [6 points] Let $N(x)$ be the cost (in dollars) to produce x pieces of chocolate. The chocolates are then put into boxes containing ten pieces of chocolate each. The packaging costs for each box of chocolates is \$0.15. Write down a mathematical expression describing the following.

a. [2 points] The average cost (in dollars per piece of chocolate) of producing c chocolates.

Solution: $\dfrac{N(c)}{c}$

b. [2 points] The cost in dollars of producing the fifteenth piece of chocolate.

Solution: $N(15) - N(14)$

c. [2 points] The total cost in dollars (including packaging costs) of producing b boxes of chocolate.

Solution: $0.15b + N(10b)$.