4. [8 points] The number of people p (in thousands) who are sick with the flu virus t days after January 1, 2014 is given by

$$p = g(t) = \frac{3}{1 + e^{-0.3t}}$$

a. [4 points] Find a formula for $g^{-1}(p)$. Show all your steps to receive full credit.

Solution:

$$\begin{aligned} p &= \frac{3}{1 + e^{-0.3t}} \\ p(1 + e^{-0.3t}) &= 3 \\ 1 + e^{-0.3t} &= \frac{3}{p} \\ e^{-0.3t} &= \frac{3}{p} - 1 \\ -0.3t &= \ln\left(\frac{3}{p} - 1\right) \\ t &= \frac{1}{-0.3}\ln\left(\frac{3}{p} - 1\right) \end{aligned}$$

b. [2 points] What is a practical interpretation of $g^{-1}(2)$? You do not need to compute its value. Include units.

Solution: It is the number of days after January 1, 2014 needed for two thousand individuals to be sick with the flu.

c. [2 points] The quantity of flu vaccine q (in liters) produced t days after January 1, 2014 is given by

$$q = f(t) = \frac{\sqrt{5}\, t^2}{(1 + 2t)^2}.$$

What eventually happens to the quantity of flu vaccine produced. Give your answer in **exact form**.

Solution: $\lim_{t\to\infty} f(t) = \lim_{t\to\infty} \frac{\sqrt{5}\, t^2}{(1+2t)^2} = \lim_{t\to\infty} \frac{\sqrt{5}\, t^2}{(2t)^2} = \lim_{t\to\infty} \frac{\sqrt{5}\, t^2}{4t^2} = \frac{\sqrt{5}}{4}.$