

7. [9 points] The total cost C (in thousands of dollars) for a farmer to grow p tons of potatoes is given by the function

$$C = h(p) = 2p^2 - 16p + 39.$$

- a. [4 points] What is the maximum number of tons of potatoes the farmer can produce if he only has 35 thousand dollars to spend on growing potatoes? Your answer should be in **exact form**.

Solution:

$$2p^2 - 16p + 39 = 35$$

$$2p^2 - 16p + 4 = 0$$

$$p^2 - 8p + 2 = 0$$

$$p = \frac{8 \pm \sqrt{64 - 4(2)}}{2} = \frac{8 \pm \sqrt{56}}{2} = 4 \pm \sqrt{14}$$

Number of tons of potatoes = $4 + \sqrt{14}$.

- b. [4 points] Complete the square to write the function h in vertex form. Show all your work step by step.

Solution:

$$2p^2 - 16p + 39 = 2(p^2 - 8p) + 39$$

$$= 2(p^2 - 8p + 16 - 16) + 39$$

$$= 2((p - 4)^2 - 16) + 39$$

$$= 2(p - 4)^2 - 32 + 39 = 2(p - 4)^2 + 7$$

- c. [1 point] How many tons of potatoes does the farmer need to produce in order to minimize the total cost?

Solution: Number of tons of potatoes = 4.