7. [9 points] The total cost $C$ (in thousands of dollars) for a farmer to grow $p$ tons of potatoes is given by the function

$$C = h(p) = 2p^2 - 16p + 39.$$

a. [4 points] What is the maximum number of tons of potatoes the farmer can produce if he only has 35 thousand dollars to spend on growing potatoes? Your answer should be in exact form.

**Solution:**

\[
2p^2 - 16p + 39 = 35 \\
2p^2 - 16p + 4 = 0 \\
p^2 - 8p + 2 = 0 \\
p = \frac{8 \pm \sqrt{64 - 4(2)}}{2} = 4 \pm \sqrt{14}
\]

Number of tons of potatoes = $4 + \sqrt{14}$.

b. [4 points] Complete the square to write the function $h$ in vertex form. Show all your work step by step.

**Solution:**

\[
2p^2 - 16p + 39 = 2(p^2 - 8p) + 39 \\
= 2(p^2 - 8p + 16 - 16) + 39 \\
= 2((p - 4)^2 - 16) + 39 \\
= 2(p - 4)^2 - 32 + 39 = 2(p - 4)^2 + 7
\]

c. [1 point] How many tons of potatoes does the farmer need to produce in order to minimize the total cost?

**Solution:** Number of tons of potatoes = 4.