8. [9 points] A space ship has landed on Planet X. Scientists discovered that the surface temperature of Planet X oscillates sinusoidally between a maximum of $170°C$ to a minimum of $-40°C$. It takes 7 hours for the surface temperature to decrease from its maximum to its minimum. At the time the space ship landed, the surface temperature was $-40°C$. Let $P = g(t)$ be the surface temperature (in °C) of Planet X, t hours after the space ship landed.

a. [4 points] Find a formula for $g(t)$.

Solution: Maximum temperature=170°C and Minimum temperature=−40°C, hence

1.Midline: $P = k = \frac{Max+Min}{2} = \frac{170+(-40)}{2} = 65$.

2.Period=T=2(7)=14 hours.

3.Amplitude=$A = \frac{Max-Min}{2} = \frac{170-(-40)}{2} = 105$

4.Since the graph has a minimum at $t = 0$, then the function is given by a negative cosine cycle.

Hence using

$$g(t) = k - A\cos\left(\frac{2\pi}{T}t\right) = 65 - 105\cos\left(\frac{2\pi}{14}t\right) = 65 - 105\cos\left(\frac{\pi}{7}t\right)$$

The surface temperature K (in °C) of a moon of Planet X, t hours after the spaceship landed on Planet X, is given by the formula

$$K = Q(t) = 20 - 70\cos\left(\frac{2\pi}{3}t\right).$$

b. [5 points] Find the times in the interval $-1 \le t \le 3$ when the surface temperature of the moon is equal to 10° C. Your solutions should be in **exact form**.

Solution:

$$20 - 70\cos\left(\frac{2\pi}{3}t\right) = 10$$

$$\cos\left(\frac{2\pi}{3}t\right) = \frac{1}{7}$$

$$\frac{2\pi}{3}t = \cos^{-1}\left(\frac{1}{7}\right)$$

$$t = \frac{3}{2\pi}\cos^{-1}\left(\frac{1}{7}\right)$$

Using the symmetries of the cosine function we get that in the interval $-1 \le t \le 3$, there are three solutions to the equation $t_1 < t_2 < t_3$.

$$t_2 = \frac{3}{2\pi}\cos^{-1}\left(\frac{1}{7}\right) \qquad t_1 = -t_2 \qquad t_3 = 3 - t_2.$$