

10. [10 points]

- a. [5 points] Find all the values of  $-4 \leq x \leq 20$  that satisfy the following equation. Find your answers algebraically. Your answer(s) must be in **exact form**. Show all your work.

$$2 - 6 \sin\left(\frac{\pi}{8}x\right) = 4$$

*Solution:*

$$\begin{aligned} 2 - 6 \sin\left(\frac{\pi}{8}x\right) &= 4 \\ \sin\left(\frac{\pi}{8}x\right) &= -\frac{1}{3} \\ \frac{\pi}{8}x &= \sin^{-1}\left(-\frac{1}{3}\right) \\ x_1 &= \frac{8}{\pi} \sin^{-1}\left(-\frac{1}{3}\right) \end{aligned}$$

$$x_1 = x_1$$

$$x_2 = 8 - x_1$$

$$x_3 = 16 + x_1.$$

- b. [5 points] Let  $w = F(s)$ , where  $F(s) = 4 + \ln(3^s + 1)$ . Find a formula for  $F^{-1}(s)$ . Show all your work.

*Solution:*

$$\begin{aligned} w &= 4 + \ln(3^s + 1) \\ w - 4 &= \ln(3^s + 1) \\ e^{w-4} &= 3^s + 1 \\ e^{w-4} - 1 &= 3^s \\ \ln(e^{w-4} - 1) &= s \ln(3) \\ s &= \frac{\ln(e^{w-4} - 1)}{\ln(3)} = F^{-1}(w). \end{aligned}$$