

7. [10 points] The following table gives some values of the two functions $f(x)$ and $g(x)$.

x	2	8
$f(x)$	20	160
$g(x)$	4	12

- a. [5 points] Suppose the function $f(x)$ is an exponential function. Find a formula for $f(x)$. Your answer must be in **exact form**. Show all your work.

Solution:

$$ab^8 = 160$$

$$ab^2 = 20$$

$$\frac{ab^8}{ab^2} = \frac{160}{20} \quad b^6 = 8 \quad \text{hence} \quad b = 8^{\frac{1}{6}}$$

$$20 = a8^{\frac{2}{6}} = 2a \quad \text{then} \quad a = 10.$$

$$f(x) = 10(8)^{\frac{x}{6}}$$

- b. [5 points] Suppose that $g(x)$ is a power function. Find a formula for $g(x)$. Your answer must be in **exact form**. Show all your work.

Solution:

$$k(8^p) = 12$$

$$k(2^p) = 4$$

$$\frac{k8^p}{k2^p} = \frac{12}{4} = 3 \quad \text{then} \quad 4^p = 3.$$

$$\ln(4^p) = \ln(3) \quad \text{implies} \quad p = \frac{\ln(3)}{\ln(4)}$$

$$4 = k2^{\frac{\ln 3}{\ln 4}} \quad \text{yields} \quad k = \frac{4}{2^{\frac{\ln 3}{\ln 4}}}$$

$$g(x) = \frac{4}{2^{\frac{\ln 3}{\ln 4}}} x^{\frac{\ln 3}{\ln 4}}.$$