**9**. [12 points] Jemma and Sarah want to design a website for the winter sale of the store Fritz. The sale will start at 8 am and close at 8 pm on December 23. To build the website, they have to be able to predict the number of online customers that day. Each one has different predictions for the number of online customers that day.

**a**. [6 points] Sarah believes that the number of online customers will start at a minimum of 2 thousand online customers at 8 am and then it will increase to a maximum of 12 thousand customers at 2 pm. Let $S(t)$ be the sinusoidal function which gives the amount of online customers on the website (in thousands) *$t$ hours after 8 am* on December 23 according to Sarah's predictions.

i) What are the amplitude and the midline of $S(t)$?

*Solution:* Amplitude=5 Midline: $y = 7$.

ii) Find a formula for the function $S(t)$ for $0 \leq t \leq 12$.

*Solution:* The amplitude is 5, the midline is $y = 7$ and the period is 12 hours. Since $S(t)$ has a minimum at $t = 0$, we get that

$$S(t) = -5\cos\left(\frac{\pi}{6}t\right) + 7$$

(*The statement of the problem has been included below for your own convenience*)
Jemma and Sarah want to design a website for the winter sale of the store Fritz. The sale will start at 8 am and close at 8 pm on December 23. To build the website, they have to be able to predict the number of online customers that day. Each one has different predictions for the number of online customers that day.

**b**. [6 points] On the other hand, Jemma believes that there will be 3 thousand online customers at 8 am. She expects that the number of online customers will reach a maximum of 10 thousands at 2 pm.
Let $J(t)$ be the quadratic function which gives the amount of online customers on the website (in thousands) *$t$ hours after 8 am* on December 23 according to Jemma's predictions.

i) What is the vertex of $J(t)$?

*Solution:* The vertex is (6,10) since maximum is at 2 pm ($t = 6$) with 10 thousands online customers.

ii) Find a formula for $J(t)$ for $0 \leq t \leq 12$.

*Solution:* The vertex is (6,10). So the quadratic function $J$ in vertex form is given by

$$J(t) = a(t-6)^2 + 10.$$

Since $J(0) = 3$, we have that $3 = a(-6)^2 + 10$ and $a = -\frac{7}{36}$. Thus

$$J(t) = -\frac{7}{36}(t-6)^2 + 10.$$