1. [8 points] Radioactive waste has been draining into a lake, causing trees in the surrounding forest to wilt and die. On March 24, 2010, local scientists surveyed the surrounding forest, determining that $S(m)$ thousand trees had begun to wilt m kilometers out from the center of the lake. The scientists determined that 20 thousand trees were decaying 20 kilometers out, and 5 thousand trees were decaying 60 kilometers out.

a. [3 points] Assume that the function S is invertible. Find the average rate of change of the inverse function S^{-1} on the interval $[5, 20]$. Write your final answer *in the space provided*, and **include units**.

Solution: The average rate of change is

$$\frac{S^{-1}(20) - S^{-1}(5)}{20-5} = \frac{20-60}{20-5} = -\frac{40}{15}$$

The average rate of change of S^{-1} on the interval $[5, 20]$ is $-\frac{40}{15}$ kilometers/thousand trees

b. [5 points] Find a formula for $S(m)$ in terms of m, assuming that S is a power function of m. Your answer should be **exact**, and you must **show your work** carefully, writing your final answer *in the space provided.*

Solution: If $S(m)$ is a power function, it must have formula am^b for some constants a, b. We know that $S(20) = 20$ and $S(60) = 5$. This gives us:

$$a(20)^b = 20$$
$$a(60)^b = 5$$

If we divide the first equation by the second, we get:

$$\frac{(20)^b}{(60)^b} = \frac{20}{5}$$

which simplifies to:

$$\left(\frac{1}{3}\right)^b = 4$$

Taking the natural logarithm of both sides of this equation and simplifying gives us:

$$b \ln\left(\frac{1}{3}\right) = \ln 4$$
$$b = \frac{\ln 4}{\ln\left(\frac{1}{3}\right)}$$
$$b = -\frac{\ln 4}{\ln 3}$$

And, finally, plugging this into the first equation gives us:

$$a(20)^{-(\ln 4)/(\ln 3)} = 20$$
$$a = \frac{20}{20^{-(\ln 4)/(\ln 3)}}$$

$S(m) =$ $\frac{20}{20^{-(\ln 4)/(\ln 3)}} m^{-(\ln 4)/(\ln 3)}$