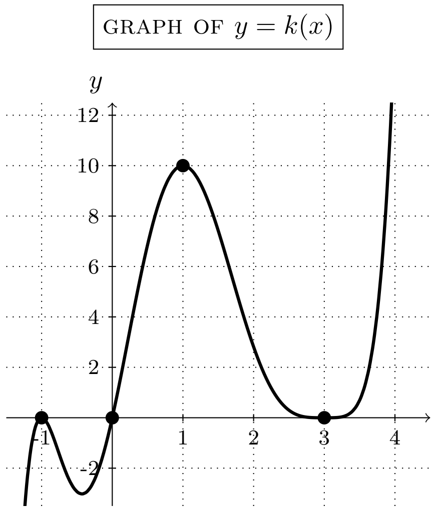
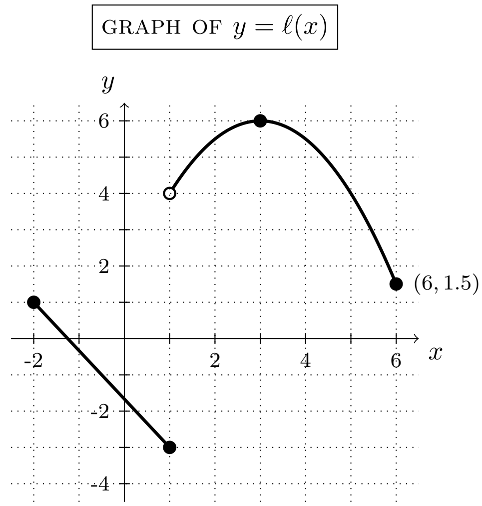
**10**. [11 points] Consider the graphs of $y = k(x)$ and $y = \ell(x)$ given below:




You must **show your work** in both parts of this problem to receive full credit. Write your final answers *in the spaces provided.*

**a**. [5 points] Find a formula for $k(x)$, assuming $k(x)$ is a polynomial of degree seven with zeros at $x = -1$, $x = 0$ and $x = 3$.

**Solution**: The zeros of $k(x)$ are $x = -1$, 0 and 3. From the graph, we can see that the zero at $x = 0$ has multiplicity one, and the zeroes at $x = -1$ and $x = 3$ have even multiplicities. The graph is clearly flatter at $x = 3$, so the multiplicity at this zero is larger than the multiplicity of the zero at $x = -1$. Since we know $k(x)$ has degree 7, we know that the multiplicity of the zero at $x = -1$ must be 2, and the multiplicity at $x = 3$ must be 4, giving us $k(x) = ax(x+1)^2(x-3)^4$ for some constant $a$.
Since the point $(1, 10)$ is on the graph, we know that we must have $k(1) = 10$, and hence $10 = a(2)^2(-2)^4$, giving us $a = \frac{5}{32}$.

$k(x) =$ $\frac{5}{32}x(x+1)^2(x-3)^4$

**b**. [6 points] Find a piecewise-defined formula for $\ell(x)$ on $[-2, 6]$, given that the graph of $y = \ell(x)$ is made up of a line and a parabola.

**Solution**: We see from the graph that the linear part has slope

$$\frac{-3-1}{1-(-2)} = -\frac{4}{3}$$

So for the linear part, we must have $\ell(x) - (-3) = -\frac{4}{3}(x-1)$, and hence $\ell(x) = -\frac{4}{3}(x-1) - 3$. The quadratic part clearly has a vertex at $(3, 6)$, and so we must have $\ell(x) = a(x-3)^2 + 6$ for some constant $a$. Since the point $(6, 1.5)$ is on the quadratic part, we get $1.5 = a(3)^2 + 6$, and so $a = -0.5$.

$$\ell(x) = \begin{cases} -\frac{4}{3}(x-1) - 3 & \text{if } -2 \le x \le 1 \\ -0.5(x-3)^2 + 6 & \text{if } 1 < x \le 6 \end{cases}$$