2. [7 points] Olga runs a factory that produces pitch, and finds that the cost C (in thousands of dollars) to produce g gallons of pitch is given by $C = f(g)$, where:

$$f(g) = 5 + \log\left(3 + e^{7g}\right)$$

for $g \geq 0$. Note that f is an invertible function.

a. [5 points] Find a formula for the quantity of pitch $f^{-1}(C)$ (in gallons) that the factory must have produced in terms of the total cost C (in thousands of dollars) incurred. You must **show your work** carefully for this part.

Solution: We solve for g in the equation given, first subtracting 5:

$$\begin{aligned} C &= 5 + \log\left(3 + e^{7g}\right) \\ \log\left(3 + e^{7g}\right) &= C - 5 \end{aligned}$$

We exponentiate both sides to remove the natural logarithm:

$$\begin{aligned} 3 + e^{7g} &= 10^{(C-5)} \\ e^{7g} &= 10^{(C-5)} - 3 \end{aligned}$$

And we now take a natural logarithm to isolate g:

$$\begin{aligned} 7g &= \ln\left(10^{(C-5)} - 3\right) \\ g &= \frac{1}{7}\ln\left(10^{(C-5)} - 3\right). \end{aligned}$$

$f^{-1}(C) =$ $\frac{1}{7}\ln\left(10^{(C-5)} - 3\right)$

b. [2 points] What is the range of $f^{-1}(C)$? Write your final answer *in the space provided*, **using inequalities**.

The range of $f^{-1}(C)$ *is* $g \geq 0$