4. [8 points] In this problem, you should **show your work**. All your answers should be **exact**, and must be found *algebraically*. Write your final answers *in the spaces provided*.

For parts (a) and (b), consider the function

$$F(x) = \frac{(100x^2+3)(x^2+2x-1)}{(x^2-2x-3)(2x^2+4)}$$

a. [2 points] Find the horizontal intercept(s) of $y = F(x)$. If the function has no horizontal intercepts, write NONE in the space provided.

Solution: To find the horizontal intercept of $y = F(x)$, we need to find all values of x for which $(100x^2+3)(x^2+2x-1) = 0$. This means that either $100x^2+3 = 0$ or $x^2+2x-1 = 0$, and the first equation has no solutions since $100x^2+3$ is always at least 3. To solve $x^2+2x-1 = 0$, we use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{2^2-4(-1)}}{2} = \frac{-2 \pm \sqrt{8}}{2} = -1 \pm \sqrt{2}.$$

We're almost done, but we need to make sure that the denominator, $(x^2-2x-3)(2x^2+4)$ isn't zero when $x = -1 \pm \sqrt{2}$. We could just plug these values of x into the denominator and check (with a calculator) that we don't get 0, or notice that the denominator can be factored as $(x-3)(x+1)(2x^2+4)$, and so the denominator can only be zero for $x = 3$ or $x = -1$ (and hence, cannot be zero for $x = -1 \pm \sqrt{2}$).

Horizontal intercept(s): $\left(-1+\sqrt{2}, 0\right)$ and $\left(-1-\sqrt{2}, 0\right)$

b. [2 points] Find the equation(s) of the horizontal asymptote(s) of $y = F(x)$. If the function has no horizontal asymptotes, write NONE in the space provided.

Solution: The leading term in the numerator is $100x^4$, and the leading term in the denominator is $2x^4$. So:

$$\lim_{x\to\infty} F(x) = \lim_{x\to\infty} \frac{100x^4}{2x^4} = 50$$

Horizontal asymptote(s): $y = 50$

c. [4 points] Consider the function

$$G(x) = \frac{x^2(x^2+5)^3}{(x-2)(x^2+5)^4 x}$$

Find the equation(s) of the vertical asymptote(s) of $y = G(x)$, and the x-coordinate(s) of the hole(s) of $y = G(x)$. If the function has no vertical asymptotes or has no holes, write NONE in the relevant space.

Solution: Note that (x^2+5) is always positive, so we don't need to consider it when finding the holes and the vertical asymptotes. It's then easy to see that $y = G(x)$ has a hole at $x = 0$, and a vertical asymptote at $x = 2$.

Vertical asymptote(s): $x = 2$

x-coordinate(s) of hole(s): 0