6. [11 points] For this problem, your final answers must be **exact** and should be written *in the spaces provided*.

a. [5 points] Let $V(t)$ be the voltage across a resistor in a circuit (measured in volts) t minutes after 8:00 a.m. on January 29, 2013. The function $V(t)$ is periodic, and it takes 5 minutes to go from a minimum of -10 volts to a maximum of 40 volts. At 8:37 a.m., the voltage across the resistor is -10 volts. Find a formula for $V(t)$, assuming $V(t)$ is a sinusoidal function of t.

Solution: The maximum of $V(t)$ is 40 volts and the minimum -10 volts, so we can calculate the amplitude and the midline:

$$\text{Amplitude} = \frac{40-(-10)}{2} = 25$$

and

$$\text{Midline} : y = \frac{40+(-10)}{2} = 15$$

Since it takes 5 minutes to go from a minimum to a maximum, we know the period is $2 \cdot 5 = 10$. And finally, since the point $(37, -10)$ is a minimum on the graph of $V(t)$, we get:

$$V(t) = -25\cos\left(\frac{2\pi}{10}(t-37)\right) + 15$$

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b. [6 points] Find all values of t in the interval $-0.5 \le t \le 1$ for which:

$$5\sin\left(2\pi\left(t+\frac{1}{4}\right)\right) + 3 = 0$$

Your answer must be found *algebraically* and should be **exact**. You must **show your work** carefully to receive full credit.

Solution: We first isolate the sine to get:

$$5\sin\left(2\pi\left(t+\frac{1}{4}\right)\right) + 3 = 0$$
$$\sin\left(2\pi\left(t+\frac{1}{4}\right)\right) = -0.6$$

Two 'different' solutions to $\sin(x) = -0.6$ are given by $x = \sin^{-1}(-0.6)$ and $x = \pi - \sin^{-1}(-0.6)$, and so we can get two 'different' solutions to the equation above by setting:

$$\begin{aligned} 2\pi(t+0.25) &= \sin^{-1}(-0.6) \\ t+0.25 &= \frac{\sin^{-1}(-0.6)}{2\pi} \\ t &= \frac{\sin^{-1}(-0.6)}{2\pi} - 0.25 \end{aligned}$$

$$\begin{aligned} 2\pi(t+0.25) &= \pi - \sin^{-1}(-0.6) \\ t+0.25 &= 0.5 - \frac{\sin^{-1}(-0.6)}{2\pi} \\ t &= 0.25 - \frac{\sin^{-1}(-0.6)}{2\pi} \end{aligned}$$

Finally, we need to add/subtract the period to get the solutions in the interval $[-0.5, 1]$. Doing this with a calculator gives us our final answer (below).

The solutions in $-0.5 \le t \le 1$ *are* $\frac{\sin^{-1}(-0.6)}{2\pi} - 0.25; -\frac{\sin^{-1}(-0.6)}{2\pi} + 0.25; \frac{\sin^{-1}(-0.6)}{2\pi} + 0.75$