

7. [8 points] Some values of the function $f(x)$ are given in the table below.

x	-2	0	2	5	8	10	15	18	20	21
$f(x)$	54.22	30.50	17.16	7.24	13.84	18.24	29.24	5	-9	-20

Note that all values in the table have been rounded to two decimal places. You must **show your work** for each part of this problem, and write your final answers *in the spaces provided*.

- a. [3 points] Find a formula for $f(x)$ valid for $-2 \leq x \leq 5$, assuming that f is exponential on the interval $[-2, 5]$.

Solution: Since $f(x)$ is exponential, we know that it has the form $f(x) = ab^x$. From the table above, we see immediately that $a = f(0) = 30.50$. To get b , we use the fact that $f(2) = 17.16$ to get:

$$\begin{aligned} 30.5b^2 &= 17.16 \\ b^2 &= \frac{17.16}{30.5} \\ b &= \sqrt{\frac{17.16}{30.5}} \end{aligned}$$

$$f(x) = \underline{30.5 \left(\sqrt{\frac{17.16}{30.5}} \right)^x}$$

- b. [2 points] Find a formula for $f(x)$ valid for $5 \leq x \leq 15$, assuming that $f(x)$ is linear on the interval $[5, 15]$.

Solution: Since $f(x)$ is linear, it has the form $f(x) = mx + b$. We find the slope using the points $(5, 7.24)$ and $(8, 13.84)$:

$$m = \frac{13.84 - 7.24}{8 - 5} = \frac{6.6}{3} = 2.2.$$

So we get $f(x) - 7.24 = 2.2(x - 5)$, and hence $f(x) = 2.2(x - 5) + 7.24$.

$$f(x) = \underline{2.2(x - 5) + 7.24}$$

- c. [3 points] Show that $f(x)$ cannot be concave down on the interval $[15, 21]$. Make sure any relevant calculations are clearly shown, and write a *brief* sentence explaining your reasoning.

Solution: Calculating the average rate of change of $f(x)$ on the intervals $[15, 18]$ and $[18, 20]$ gives us:

$$\frac{5 - 29.24}{18 - 15} = \frac{-24.24}{3} = -8.08$$

and

$$\frac{-9 - 5}{20 - 18} = \frac{-14}{2} = -7.$$

Since the average rate of change on $[18, 20]$ is greater than the average rate of change on $[15, 18]$, $f(x)$ cannot be concave down on $[15, 21]$.