8. [15 points] The number of hemlock trees in the southern Appalachian mountains is declining as a result of an infestation of hemlock woolly adelgids (a kind of insect).

- There are $H(d)$ *healthy* hemlock trees in the southern Appalachian mountains d days after January 1, 2013.
- There are $I(d)$ *infested* hemlock trees in the southern Appalachian mountains d days after January 1, 2013.

Note that all hemlock trees are considered healthy unless they are infested. Be sure to write your final answers *in the spaces provided*.

a. [2 points] Let $J(w)$ be the number of *healthy* hemlock trees in the southern Appalachian mountains w *weeks* after January 1, 2013. Find a formula for $J(w)$ in terms of the functions H or I (or possibly both).

$J(w) =$ $H(7w)$

b. [3 points] Let $F(d)$ be the fraction of the hemlock trees in the southern Appalachian mountains that are *infested* d days after January 1, 2013. Find a formula for $F(d)$ in terms of the functions H or I (or possibly both).

$F(d) =$ $\dfrac{I(d)}{H(d)+I(d)}$

c. [4 points] Let $K(d)$ be the total number of hemlock trees in the southern Appalachian mountains, in *thousands*, d days after January 1, 2013. Find a formula for $K(d)$ in terms of the functions H or I (or possibly both).

$K(d) =$ $0.001(H(d)+I(d))$

This problem continues on the next page

d. [3 points] The number of hemlock trees I that are *infested* in the southern Appalachian mountains is *inversely proportional* to the cube of the total amount of money M (in millions of dollars) that the government spends combating the spread of the adelgids. Write a formula for I in terms of M, assuming that there were 2,000 infested trees when the government had spent 3 million dollars. You must **show your work** for this part.

Solution: Since I is inversely proportional to the cube of M, we have:

$$I = \frac{k}{M^3}$$

We know that $I = 2000$ when $M = 3$, which gives us:

$$2{,}000 = \frac{k}{3^3}$$

and so we have $k = 3^3 \cdot 2{,}000 = 162{,}000$.

$I =$ $\frac{162,000}{M^3}$

e. [3 points] The number of hemlock woolly adelgids $A(M)$ (in millions) is also a function of the amount of money M (in millions of dollars) that the government spends to try to preserve the hemlock trees, and is given by:

$$A(M) = \frac{4}{M}$$

for $M \geq 4$. Find the equation of the horizontal asymptote of $y = A(M)$, **and** interpret this horizontal asymptote in practical terms.

Solution: It's easy to see that the horizontal asymptote is $y = 0$. Practically, this means that as the amount of money the government spends increases (going to ∞), the number of hemlock woolly adelgids in the southern Appalachian mountains approaches 0.

The equation of the horizontal asymptote is $y = 0$