**9.** [7 points] Consider the circle of radius R centered at the point O, illustrated below. Note that the diagram is not drawn to scale.



Note that the line AB contains the point O, and the angles ACB and ADC both have measure  $\frac{\pi}{2}$  radians.  $\alpha$  is the positive measure of the angle COD (see the dagram), while L is the length of the line segment AC.

**a**. [2 points] Find the length of the line segment CD. Your answer for this part may involve any or all of the constants R, L and  $\alpha$ .

The length of CD is  $R \sin \alpha$ 

**b.** [3 points] Find the (positive) measure of the angle OAC in radians. Your answer for this part may involve the constants R and L, but must **not** include the constant  $\alpha$ .

Solution: Let  $\beta$  be the measure of the angle OAC in radians. Note that AB has length 2R, so in the right triangle ACB, we have:

and hence

$$\cos \beta = \frac{1}{2R}$$
$$\beta = \cos^{-1} \left(\frac{L}{2R}\right)$$

The measure of OAC is  $\frac{\cos^{-1}\left(\frac{L}{2R}\right)}{2R}$ 

c. [2 points] Find the length of the (bolded) circular arc AC. Your answer for this part may involve any or all of the constants R, L and  $\alpha$ .

The length of the arc AC is  $R(\pi - \alpha)$