

1. [8 points] For both parts of this question, please leave your answers in **exact form** and show **all** your work.

- a. [3 points] Find all values of w satisfying the following equation.

$$\ln(10w + 3) = 8.$$

Solution: Take the base e exponential of both sides to get

$$10w + 3 = e^8.$$

We can then solve this for w as follows:

$$10w = e^8 - 3$$

$$w = \frac{e^8 - 3}{10}$$

$$w = \frac{e^8 - 3}{10}.$$

- b. [5 points] Find all values of x with $-2 \leq x \leq 1$ satisfying the following equation.

$$4 \tan\left(\frac{2\pi}{3}x\right) = 5.$$

Solution: To find the principal solution, we solve

$$\tan\left(\frac{2\pi}{3}x\right) = 5/4,$$

$$\frac{2\pi}{3}x = \tan^{-1}(5/4).$$

Hence,

$$x = \frac{3}{2\pi} \tan^{-1}(5/4) \approx 0.428.$$

To find the other solutions, observe that the period of the function is $\pi \cdot \frac{3}{2\pi} = 3/2$. Hence, we also have a solution at

$$x = \frac{3}{2\pi} \tan^{-1}(5/4) - \frac{3}{2}.$$

$$x = \frac{3}{2\pi} \tan^{-1}(5/4), \quad \frac{3}{2\pi} \tan^{-1}(5/4) - \frac{3}{2}.$$