2. [16 points] The five parts of this question are **unrelated** to each other.

a. [3 points] $f(x)$ is a periodic function with domain $(-\infty, \infty)$, with a period of 5, with an amplitude of 2, and with midline $y = -1$. Find the amplitude, period and midline of $g(x) = -7f(2(x-3)) + 1$.

The period of $g(x)$ is $5/2$.

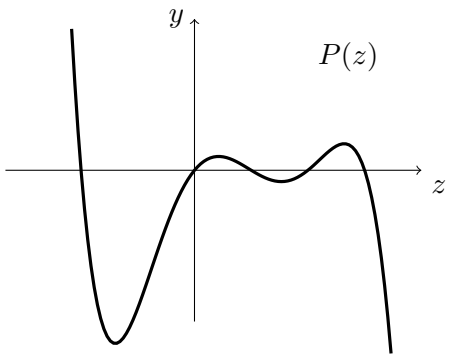
The amplitude of $g(x)$ is 14.

The midline of $g(x)$ is $y = 8$.

b. [2 points] Let $a, b, c > 0$ be positive constants. Evaluate the following limit. You do not need to show any work for this part.

$$\lim_{x\to\infty} \frac{(\sqrt{x}+2)^6}{ax^3+bx+c} = 1/a.$$

c. [3 points] The graph of a polynomial $y = P(z)$ **with its end behavior shown** is graphed below. Answer the following questions about $P(z)$.



Is the degree of $P(z)$ even or odd?

even [odd] not possible to tell

Is the leading coefficient of $P(z)$ positive or negative?

positive [negative] not possible to tell

What is the smallest possible degree of $P(z)$?

The smallest possible degree of $P(z)$ is 5.

2. (continued) Reminder: The parts of this question are **unrelated** to each other.

d. [3 points] The table below give some values of a function $M(t)$ at different t-values.

t	0	3	5
$M(t)$	625	900	1296

What type of function could $M(t)$ be? Circle all that apply.

linear exponential **quadratic** none of these

Could $M(t)$ be proportional to t^2?

yes **no**

If you answered no, briefly explain why not, and if you answered yes, find the constant of proportionality.

Solution: To be proportional to a power function, we would need $M(0) = 0$.

e. [5 points] Consider the function, $H(t) = 3e^{1.2t-3}$, that gives the weight, in grams of a mealworm t days after it hatches. Find the weight of a mealworm when it hatches, find the daily (non-continuous) growth rate of $H(t)$, and find the amount of time it takes for a mealworm to triple in weight, all in **exact form**, with **appropriate units**. Be sure to show **all** your work.

Solution: We want to find t such that $H(t) = 3H(0)$, i.e.

$$3e^{1.2t-3} = 3 \cdot 3e^{-3}.$$

Divide both sides by $3e^{-3}$ to get

$$e^{1.2t} = 3.$$

Take the logarithm both sides, and solve to get

$$1.2t = \ln 3$$

$$t = \frac{\ln 3}{1.2}.$$

The weight when it hatches is $3e^{-3}$ grams.

The daily (not continuous) growth rate is $e^{1.2} - 1$.

The time it takes to triple in weight is $\frac{\ln 3}{1.2}$ days.