3. [10 points] Desperate, Chump enlisted Chuck and Samsa's help in searching for Mrs Chump. The two of them flew high and low, and finally tracked Gregor and Mrs Chump to a cave in the side of Mt. Eggerest. Neither Chuck nor Samsa were strong enough to subdue Gregor, so they planned to rescue Mrs Chump when Gregor fell asleep.

a. [5 points] Gregor's sleepiness level (measured in *snores*) follows a 24 hour cycle. His minimum sleepiness is 2.7 snores at 9:30 pm every day, and his maximum sleepiness is 8.9 snores at 9:30 am. Let $W(t)$ be a sinusoidal function modeling Gregor's sleepiness level (in snores) t **hours after 5 pm on Friday**. Find a formula for $W(t)$.

Solution: The midline of $W(t)$ is $y = (8.9+2.7)/2 = 5.8$, its amplitude is $(8.9-2.7)/2 = 3.1$, and its period is 24. As such, we see that

$$W(t) = 3.1\cos(\frac{2\pi}{24}(t-h)) + 5.8$$

where h is some shift. To compute h, notice that 9:30 am is 16.5 hours after 5 pm, and so $W(t)$ is at a maximum at $t = 16.5$. Hence, we need to shift the graph 16.5 units to the right, and $h = 16.5$.

$W(t) =$ $3.1\cos(\frac{\pi}{12}(t-16.5)) + 5.8$.

b. [5 points] At 7 am, Chuck made his move and slipped quietly into Gregor's cave. To his horror, Gregor sensed movement and stirred awake, leaving Chuck with no choice but to inject him with Gretchken's cockroach tranquilizer. Due to Gregor's strong constitution, he did not fall immediately asleep. Instead, his sleepiness level (in snores) t **minutes** after being injected was

$$I(t) = -2\sin\left(\frac{\pi}{8}(t+2)\right) + 7.$$

How long did Chuck have to wait after the injection before Gregor's sleepiness level rose above 8 snores **for the first time**? Leave your answer in **exact** form, and include **appropriate units**.

Solution: We first solve for the principal value.

$$-2\sin\left(\frac{\pi}{8}(t+2)\right) + 7 = 8$$

$$\sin\left(\frac{\pi}{8}(t+2)\right) = -\frac{1}{2}$$

$$\frac{\pi}{8}(t+2) = \sin^{-1}(-1/2)$$

$$t = \frac{8}{\pi}\sin^{-1}(-1/2) - 2 = -\frac{10}{3}.$$

This however, is not the solution we want. We instead want to find the smallest *positive* solution. By inspecting the graph of the function, the solution we want is

$$-2 + 8 - \frac{8}{\pi}\sin^{-1}(-1/2) = 6 + \frac{4}{3} = \frac{22}{3}.$$

Chuck had to wait $\frac{22}{3}$ minutes.