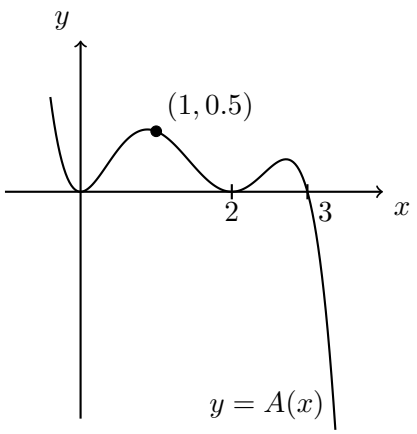
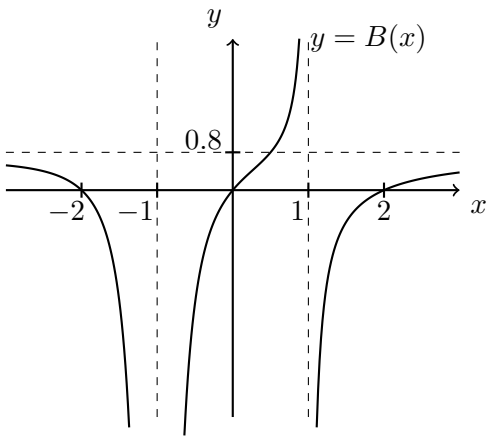
4. [11 points] Consider the graphs of $y = A(x)$ and $y = B(x)$ given below:





a. [2 points] $A(x)$ is a degree 5 polynomial. Write down all of its zeros.

$A(x)$ has zeros at $x =$ $0,\ 2,\ 3$.

b. [3 points] Write down a formula for $A(x)$, showing **all** your work.

Solution: We see that $A(x)$ has double roots at $x = 0, 2$ and a single root at $x = 3$. It thus has the formula

$$A(x) = ax^2(x-2)^2(x-3)$$

To solve for a, we plug in $x = 1$, to get

$$0.5 = a(1)^2(-1)^2(-2),$$

so $a = -\frac{1}{4}$.

$A(x) =$ $-\frac{1}{4}x^2(x-2)^2(x-3)$.

c. [3 points] The graph of $B(x)$ has vertical asymptotes at $x = -1$ and $x = 1$, and a horizontal asymptote at $y = 0.8$. If $B(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials, write down all the zeros of both polynomials.

$p(x)$ has zeros at $x =$ $-2,\ 0,\ 2$.

$q(x)$ has zeros at $x =$ $-1,\ 1$.

d. [3 points] Write down a possible formula for $B(x)$.

$B(x) =$ $\dfrac{0.8(x+2)x(x-2)}{(x+1)^2(x-1)}$.