5. [9 points] Chuck and Samsa rescued Mrs Chump and brought her back safely to Chickenville. "I'm sorry," said Chump, apologizing for the first time in his life. Seeing that the cockroaches were not the poultry-eating monsters he thought they were, Chump released all the cockroaches that had been caught. To make further amends, Chump decided to purchase dried leaves from out of town for the cockroaches to eat. t days after February 1, Chump was helping to feed N cockroaches. Some values for t and N are given in the following table.

t	4	6
N	100	1000

a. [4 points] Suppose $N = g(t)$ where $g(t)$ is an exponential function. Find a formula for $g(t)$, leaving your answer in **exact form** and showing **all** your work.

Solution: We have the general formula $g(t) = ab^t$. We solve for a and b as follows. Plugging in $t = 4$ gives

$$100 = g(4) = ab^4.$$

Plugging in $t = 6$ gives

$$1000 = g(6) = ab^6.$$

Dividing these two equations gives $b^2 = 10 \implies b = \sqrt{10}$. Now use this value of b to solve for a. We have

$$100 = a(\sqrt{10})^4 = a100$$

so $a = 1$.

$g(t) =$ $10^{t/2}$.

b. [5 points] Suppose $N = h(t)$ where $h(t)$ is a power function. Find a formula for $h(t)$, leaving your answer in **exact form** and showing **all** your work.

Solution: We have the general formula $g(t) = at^r$. We solve for a and r as follows. Plugging in $t = 4$ gives

$$100 = g(4) = a4^r$$

Plugging in $t = 6$ gives

$$1000 = g(6) = a6^r$$

Dividing these two equations gives $(6/4)^r = 10$. We take logarithms of both sides, and solve to get

$$r \log(3/2) = \log 10 = 1$$

so $r = 1/\log(3/2)$. Now use this value of r to solve for a. We have

$$100 = a4^{1/\log(3/2)}$$

$$a = 100 \cdot 4^{-1/\log(3/2)}.$$

$h(t) =$ $100 \cdot 4^{-1/\log(3/2)} t^{1/\log(3/2)}$.