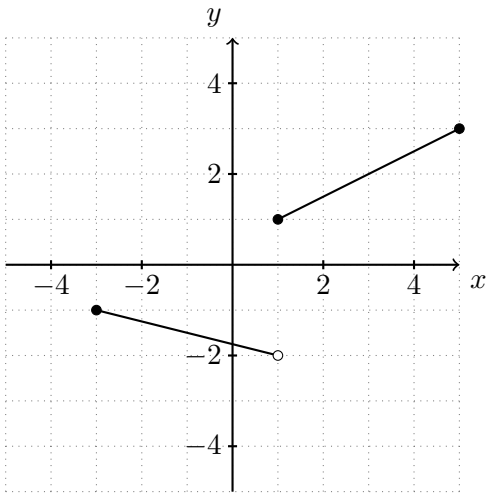
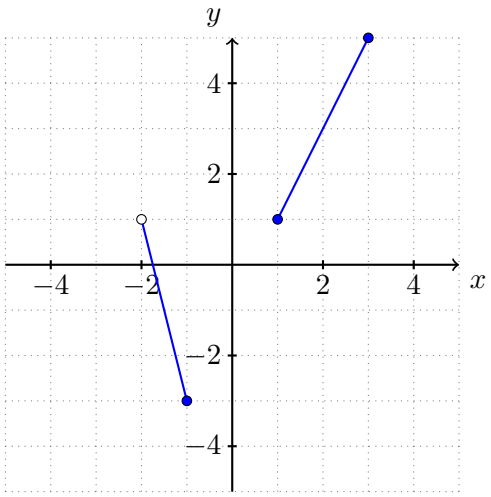
6. [11 points] The graph of $y = F(x)$ is shown below on the left.





a. [4 points] Draw the graph of $y = F^{-1}(x)$ on the provided axes.

b. [4 points] Write a piecewise formula for $F(x)$. (NOT for $F^{-1}(x)$)

Solution: The first segment has slope $\frac{(-2)-(-1)}{1-(-3)} = -\frac{1}{4}$. It passes through the point $(-3, -1)$. Hence, it is a portion of the graph of $y = -\frac{1}{4}(x+3) - 1$. The second segment has slope $\frac{3-1}{5-1} = \frac{1}{2}$. It passes through the point $(1, 1)$. Hence, it is a portion of the graph of $y = \frac{1}{2}(x-1) + 1$.

$$F(x) = \begin{cases} -\frac{1}{4}(x+3) - 1 & \text{for } -3 \le x < 1 \\ \frac{1}{2}(x-1) + 1 & \text{for } 1 \le x \le 5 \end{cases}$$

c. [3 points] Let $G(w) = \sqrt{w+1}$ with domain $(0, 8)$, and let $H(w) = F(G(w))$. What is the range for $H(w)$ and $H^{-1}(y)$? Express your answers using **interval notation**.

The range for $H(w)$ is $(1, 2)$.

The range for $H^{-1}(y)$ is $(0, 8)$.