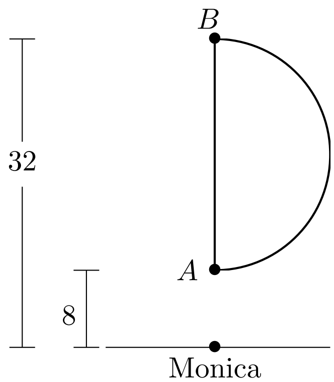
11. [10 points] Chandler wants to lose some weight after Thanksgiving and he asks Monica to coach him. His task for today is to jog **once around** a semicircular path shown in the picture below.



Chandler starts running along the arc from point A to point B and then along the straight path back to point A. He runs at a constant speed of $\frac{2\pi}{3}$ meters per second the whole time. Monica is standing 8 meters away from point A and 32 meters away from point B.

Suppose t represents the number of seconds after Chandler began to jog.

a. [3 points] For what values of t is Chandler running along the **arc** AB? You can use interval notation or inequalities.

Solution: The length of the arc AB is $s = 12\pi$ meters. If we denote Chandler's (constant) speed as v, then the time needed to cover the arc is: $T = \frac{s}{v} = \frac{12\pi}{\frac{2\pi}{3}} = 18$ seconds.

b. [4 points] While Chandler runs along the **arc** AB, $d(t)$ is the **vertical** distance between his location and the line Monica is standing on t seconds after he started jogging. Find a formula for $d(t)$. (Note that the domain of $d(t)$ should be the t values you found in part (a).)

Solution:

The function $d(t)$ is sinusoidal. The minimum and maximum values are 8 and 32 respectively and the period is 36 (according to the answer from part a). A possible formula for $d(t)$ is $20 - 12\cos(\frac{\pi}{18}t)$.

$d(t) =$ $\underline{\quad 20 - 12\cos(\frac{\pi}{18}t) \quad}$, for $\underline{\quad 0 \quad} \le t \le \underline{\quad 18 \quad}$.

c. [3 points] While Chandler runs along the **straight path** BA, $\ell(t)$ is the **vertical** distance between Chandler and the line Monica is standing on t seconds after he started jogging. Find a formula for $\ell(t)$.

Solution: The function $\ell(t)$ has to be a linear function, since Chandler is running along a straight path with constant speed. The slope of the linear function will be $-\frac{2\pi}{3}$ since Chandler is moving from B to A. Now using the fact that Chandler's distance from the line Monica is standing on is 32 meters at $t = 18$, we can use point-slope formula and conclude that: $\ell(t) = -\frac{2\pi}{3}(t - 18) + 32$. The time Chandler needs to go from B back to A is $\frac{24}{\frac{2\pi}{3}} = \frac{36}{\pi}$, where 24 is the length of the line segment BA.

$\ell(t) =$ $\underline{\quad -\frac{2\pi}{3}(t - 18) + 32 \quad}$, for $\underline{\quad 18 \quad} \le t \le \underline{\quad 18 + \frac{36}{\pi} \quad}$.

