8. [5 points] Consider the polynomial: $G(x) = x^5 - 6x^3 + 9x$. Find the zero(s) of G. Your answer should be **exact**, and must be found *algebraically*. If there are no zeros, write NONE in the space provided:

Solution:

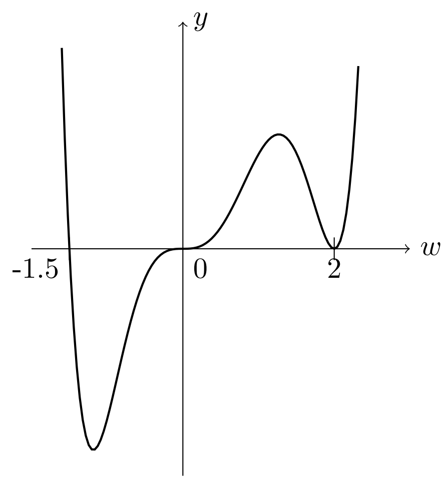
$$x^5 - 6x^3 + 9x = 0$$
$$x(x^4 - 6x^2 + 9) = 0$$

Then, $x = 0$ or

$$x^4 - 6x^2 + 9 = 0$$
$$(x^2 - 3)^2 = 0$$
$$x^2 - 3 = 0$$
$$x^2 = 3$$
$$x = \sqrt{3} \quad \text{or} \quad x = -\sqrt{3}$$

Zero(s): $0, \sqrt{3}, -\sqrt{3}$

9. [5 points] Below is part of the graph of a polynomial $P(w)$. Assume that the point (1,1.25) lies on the graph of $y = P(w)$ and $P(w)$ has **exactly** three distinct zeros. Find a possible formula for $P(w)$ so that it has the **smallest** degree possible. Show your work carefully.



Solution: We see that there are 3 distinct zeros, namely -1.5, 0 and 2, with multiplicities 1, an odd number (greater than one) and an even number respectively.

In order for the polynomial to have the smallest degree, we need to pick the smallest multiplicities possible for each zero, i.e. 1,3 and 2.

So, $P(w) = Aw^3(w + 1.5)(w - 2)^2$ for some constant A.

Since $P(1) = 1.25$ we get that $1.25 = 2.5A$ which implies that $A = \frac{1}{2}$.

$P(w) =$ $0.5w^3(w + 1.5)(w - 2)^2$