3. [10 points]
Below is the graph of a rational function $Q(x)$. Note that

- $Q(x)$ has a horizontal asymptote at $y = 0.2$
- $Q(x)$ has a hole at $x = -2$
- $Q(x)$ has zeros at $x = -4$, $x = 1$, and $x = 3$
- $Q(x)$ has vertical asymptotes at $x = -3$, $x = -1$, and $x = 2$

Using the information in the portion of the graph shown, write a possible formula for $Q(x)$. You do not need to simplify your answer.

Solution: Since there are vertical asymptotes at $x = -3$, $x = -1$ and $x = 2$, we have that we must have $(x + 3)(x + 1)(x - 2)$ in the denominator. Similarly with the zeros, we determine that $(x + 4)(x - 1)(x - 3)$ is in the numerator.

There hole at $x = -2$, but the hole is not a zero. This means that the numerator and the denominator both have equal amounts of $(x + 2)$ factors.

Finally, a horizontal asymptote at $y = 0.2$ implies that the degrees of both the numerator and denominator agree, and that the fraction of their leading coefficients is 0.2. Putting it all together, we can construct a possible formula of the form

$$Q(x) = \frac{0.2(x + 4)(x - 1)(x - 3)(x + 2)}{(x + 3)(x + 1)(x - 2)(x + 2)}$$