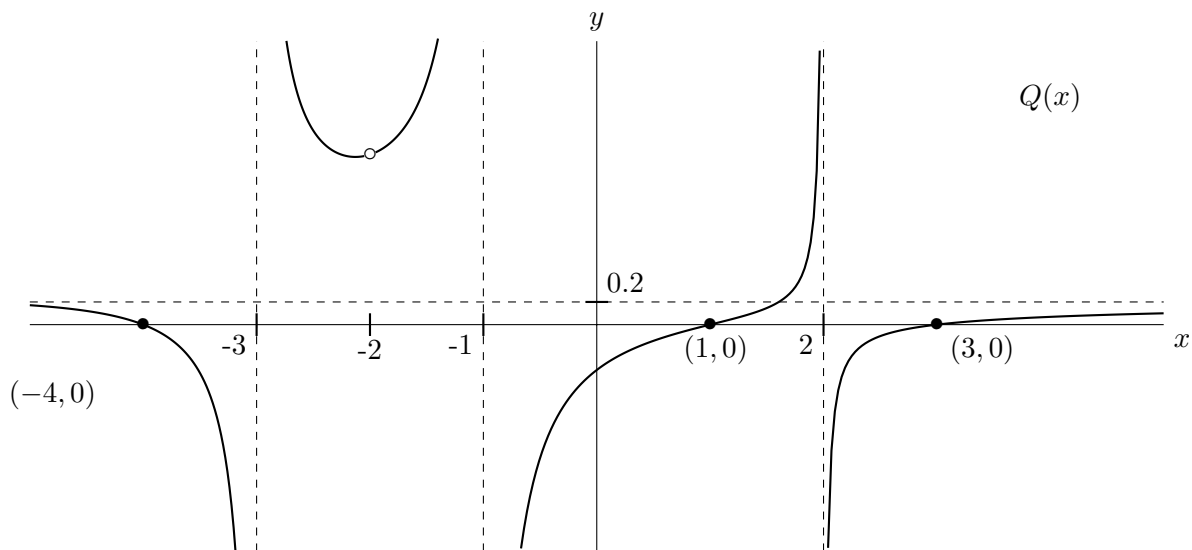


3. [10 points]

Below is the graph of a rational function  $Q(x)$ . Note that

- $Q(x)$  has a horizontal asymptote at  $y = 0.2$
- $Q(x)$  has a hole at  $x = -2$
- $Q(x)$  has zeros at  $x = -4$ ,  $x = 1$ , and  $x = 3$
- $Q(x)$  has vertical asymptotes at  $x = -3$ ,  $x = -1$ , and  $x = 2$

Using the information in the portion of the graph shown, write a possible formula for  $Q(x)$ . You do not need to simplify your answer.



*Solution:* Since there are vertical asymptotes at  $x = -3$ ,  $x = -1$  and  $x = 2$ , we have that we must have  $(x + 3)(x + 1)(x - 2)$  in the denominator. Similarly with the zeros, we determine that  $(x + 4)(x - 1)(x - 3)$  is in the numerator.

There hole at  $x = -2$ , but the hole is not a zero. This means that the numerator and the denominator both have equal amounts of  $(x + 2)$  factors.

Finally, a horizontal asymptote at  $y = 0.2$  implies that the degrees of both the numerator and denominator agree, and that the fraction of their leading coefficients is 0.2. Putting it all together, we can construct a possible formula of the form

$$Q(x) = \frac{0.2(x + 4)(x - 1)(x - 3)(x + 2)}{(x + 3)(x + 1)(x - 2)(x + 2)}$$

$Q(x) =$  \_\_\_\_\_  $Q(x) = \frac{0.2(x+4)(x-1)(x-3)(x+2)}{(x+3)(x+1)(x-2)(x+2)}$