

5. [4 points]

Let  $\alpha$  and  $\beta$  be constants such that

- $\ln(\alpha) = 2$
- $\ln(\beta) = 5$

Find the value of  $\ln(\alpha^6\beta^{-3}e^{25})$ . Your answer should **not** include  $\alpha$ ,  $\beta$ , or  $\ln$ .

*Solution:* Using log rules, we can write

$$\begin{aligned}\ln(\alpha^6\beta^{-3}e^{25}) &= \ln(\alpha^6) + \ln(\beta^{-3}) + \ln(e^{25}) \\ &= 6\ln(\alpha) - 3\ln(\beta) + 25 \\ &= 12 - 15 + 25 = 22\end{aligned}$$

$$\ln(\alpha^6\beta^{-3}e^{25}) = \underline{\hspace{2cm}22\hspace{2cm}}$$

6. [6 points]

Let  $P(x)$  be a polynomial with the following properties:

- $P(x)$  only has zeros at  $x = -3, -1, 2$
- $P(x)$  has degree 4
- The graph of  $P(x)$  passes through the points  $(-4, -36)$  and  $(-2, -8)$

Find a formula for  $P(x)$ . You do not need to simplify your answer.

*Solution:*  $P(x)$  has degree 4 but only has three zeros, so one of them must be a double zero. Note that at  $x = -4$  and  $x = -2$ , the outputs are both negative, yet there is a zero between them. This implies that  $x = -3$  must be a double zero, so that our polynomial is of the form

$$P(x) = c(x + 3)^2(x + 1)(x - 2)$$

where  $c$  is some constant. To find  $c$ , we can use one of the points, say  $(-2, -8)$ . Plugging it in, we get

$$-8 = c(-2 + 3)^2(-2 + 1)(-2 - 2) = 4c$$

Hence  $c = -2$ .

$$P(x) = \underline{\hspace{2cm}-2(x + 3)^2(x + 1)(x - 2)\hspace{2cm}}$$