9. [12 points]

a. [6 points]
While searching for cryptids, Roy claims he found a secret island with crazy thermodynamic properties. According to him, the temperature on the island fluctuates in a 24 hour cycle that can be modeled by a sinusoidal function. The maximum temperature of 45° Celsius occurs at 1 p.m. every day, and the minimum temperature of −25° Celsius occurs at 1 a.m. every day. Let the sinusoidal function $C(t)$ be the temperature, in degrees Celsius, on the island t hours after 8 a.m. Find a formula for $C(t)$.

Solution: Since it fluctuates in a 24 hour cycle, we have that the period of the function is 24. Furthermore, the midline is $y = \frac{45+(-25)}{2} = 10$ and the amplitude is $\frac{45-(-25)}{2} = 35$. Thus, we have that

$$C(t) = 35\cos(\frac{2\pi}{24}(t-h)) + 10$$

for some shift h. Note that the maximum for our function is at 1 p.m, which is 5 hours after 8 a.m. Since $\cos(t)$ naturally has a maximum at $t = 0$, and we want the maximum to be at $t = 5$, we want to shift 5 to the right. Therefore, we want $h = 5$, giving us

$$C(t) = 35\cos(\frac{2\pi}{24}(t-5)) + 10$$

b. [6 points]
On the island, Roy also claims to have found a population of the elusive Megaconda! In his notes, he writes that it is clear that the population size of Megaconda population must fluctuate in a sinusoidal manner, and that there are $M(t)$ thousand Megacondas t months after his discovery. Let

$$M(t) = 13\sin\left(\frac{\pi t}{3}\right) + 25$$

Find the first two times after Roy's discovery when the Megaconda population is 18,000. Give your answers using **exact** form.

Solution: We set up the equation

$$18 = 13\sin\left(\frac{\pi t}{3}\right) + 25$$

After some algebra, we can write this as

$$\frac{-7}{13} = \sin\left(\frac{\pi t}{3}\right)$$

We can then solve for the principal value by taking $\sin^{-1}$ of both sides, giving us

$$\sin^{-1}(\frac{-7}{13}) = \frac{\pi t}{3}$$

So

$$t = \frac{3\sin^{-1}(\frac{-7}{13})}{\pi}$$

However, this value is negative, and we want the first two positive t-values.
The first positive value can be found from the principal value using symmetry, giving us

$$t = 3 - \frac{3\sin^{-1}(\frac{-7}{13})}{\pi}$$

The second positive value can be found by adding the period to the principal value, giving us

$$t = 6 + \frac{3\sin^{-1}(\frac{-7}{13})}{\pi}$$

From this, we see that the first two times the Megaconda population is $18,000$ is $3 - \frac{3\sin^{-1}(\frac{-7}{13})}{\pi}$ and $6 + \frac{3\sin^{-1}(\frac{-7}{13})}{\pi}$ months after Roy's discovery.