

4. [11 points] Dahlia is running an experiment. A weight bobs up and down on the end of a spring that is attached to the ceiling. The distance, in inches, between the spring and the ceiling t seconds after Dahlia begins recording is given by

$$h(t) = 3 \sin\left(\frac{4\pi}{3}t\right) + 8$$

- a. [3 points] Find the period of $h(t)$, and interpret your answer in the context of the problem.

Answer: Period: 3/2

Interpretation:

Solution: From one time the weight reaches its maximum distance to the next, 1.5 seconds pass.

- b. [2 points] Which of the following best describes the spring's motion at the moment Dahlia begins recording? Choose the **one** best answer.

When Dahlia begins recording...

- i. the spring is at its average distance from the ceiling, and is moving away from the ceiling.
 - ii. the spring is at its average distance from the ceiling, and is moving toward the ceiling.
 - iii. the spring is at its farthest point from the ceiling.
 - iv. the spring is at its closest point to the ceiling.
 - v. none of these
- c. [6 points] Find the first **two** times the spring is exactly 9 inches from the ceiling. Show all of your work, and give your answers in exact form or correct to at least two decimal places. Include units.

Solution: We find one solution by solving using arcsin:

$$\begin{aligned} 3 \sin\left(\frac{4\pi}{3}t\right) + 8 &= 9 \\ 3 \sin\left(\frac{4\pi}{3}t\right) &= 1 \\ \sin\left(\frac{4\pi}{3}t\right) &= 1/3 \\ \frac{4\pi}{3}t &= \arcsin(1/3) \\ t &= \frac{3}{4\pi} \arcsin(1/3) \end{aligned}$$

Since the period is $3/2$, the second solution is $3/4 - \frac{3}{4\pi} \arcsin(1/3)$