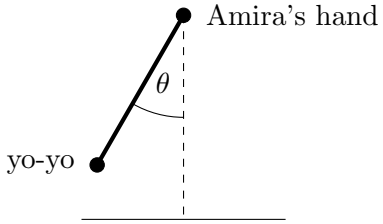
6. [10 points] Amira is using a yo-yo as a pendulum by holding the string and letting the yo-yo swing back and forth in a plane—that is, just left to right, not making any kind of ellipse when viewed from above. The symbol θ denotes the maximum angle the string makes with the vertical, as shown in the diagram to the right.



a. [2 points] If θ is 15° and the length of the string between Amira's hand and the yo-yo is 3 ft, what is the length of the entire arc that the yo-yo swings through as it travels left to right? *Show all work. Give your answer in exact form or rounded to at least two decimal places.*

Solution: Since $\theta = 15°$, then the angle of the swing from left to right is twice as much: 30°. This is 30/360 (or $\frac{1}{12}$) of a full rotation, so $2\pi \cdot \frac{30}{360}$ radians. Since the radius is 3ft, the total length of this arc is: $3 \cdot 2\pi \cdot \frac{30}{360} \approx 1.57$

Answer: $3 \cdot 2\pi \cdot \frac{30}{360} \approx 1.57$ ft

b. [1 point] If Amira adjusted the yo-yo so that the length of the string between her hand the yo-yo were only 1.5 feet instead, how would that change the length of the arc that the yo-yo swings through? *Show your work or explain.*

Solution: We'd have almost the same expression as above, but instead of 3 we'd have 1.5 being multiplied by $2\pi/12$. So our result would be half as long:

$$1.5 \cdot \frac{2\pi}{12} \approx 0.78$$

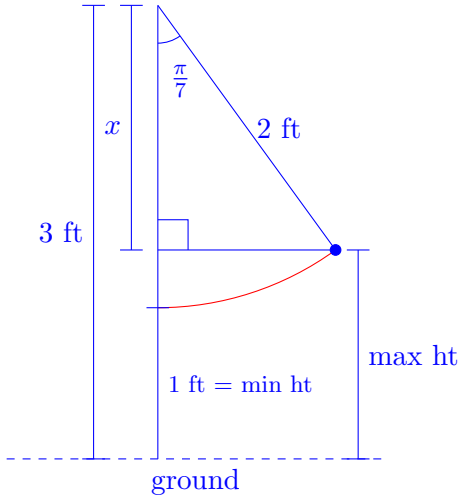
c. [7 points] Now suppose that

- the length of the string between Amira's hand and the yo-yo is 2 feet,
- at its lowest point, the yo-yo is 1 foot above the ground,
- θ is $\pi/7$ radians,
- and that it takes 1.6 seconds for the yo-yo to make a full swing from left to right and back to left again.

Give all answers below in exact form or rounded to two decimal places.

Find the maximum height of the yo-yo. *Show all work,* ***including a diagram.***

Solution: Here is a diagram depicting what is described above.



If we can find the value of marked x in the diagram, then we can find the maximum height as $3 - x$. To find x we can use

$$\cos(\pi/7) = x/2$$

That is, $x = 2\cos(\pi/7) \approx 1.80$. So the maximum height is $3 - 2\cos(\pi/7) \approx 1.2$.

Answer: $3 - 2\cos(\pi/7) \approx 1.2$ ft

Let $h(t)$ be the function giving the height, in feet, of the yo-yo at time t seconds after it is released from its maximum height. Find the amplitude and period of $h(t)$.
Include units.

Solution: We found our maximum height above. To find the amplitude we can take (max ht – min ht)/2. Since the minimum height is 1 ft (at the bottom of the pendulum swing), this is:

$$[(3 - 2\cos(\pi/7)) - 1]/2 \approx (1.2 - 1)/2 \approx 0.1$$

To find the amplitude we need to use the given fact that it takes 1.6 seconds for the pendulum to swing from left to right back to left again. However, if we are considering the *height* as a function of time, this starts repeating itself every half swing (right to left), so the period would be $1.6/2 = 0.8$ seconds.

Answer: amplitude = $((3-2\cos(\pi/7))-1)/2 \approx (1.2-1)/2 \approx 0.1$ **feet**

Answer: period = 0.8 **seconds**