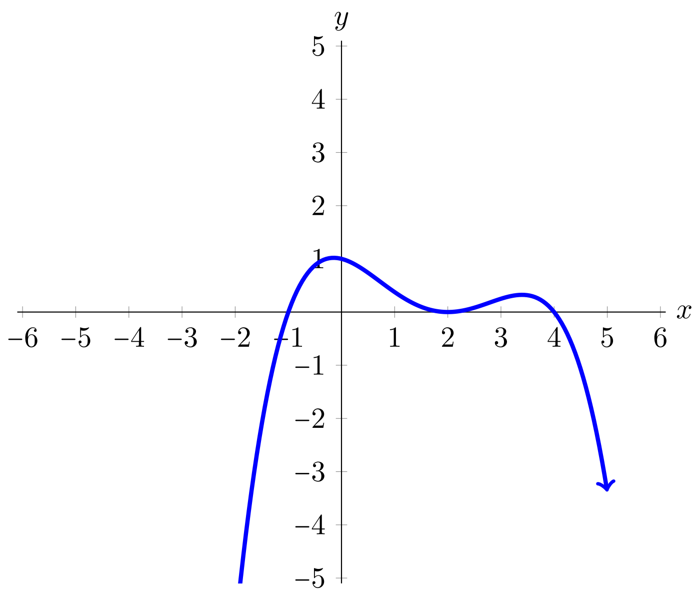
8. [8 points]

a. [4 points] Sketch a graph of a polynomial $f(x)$ satisfying the following conditions:

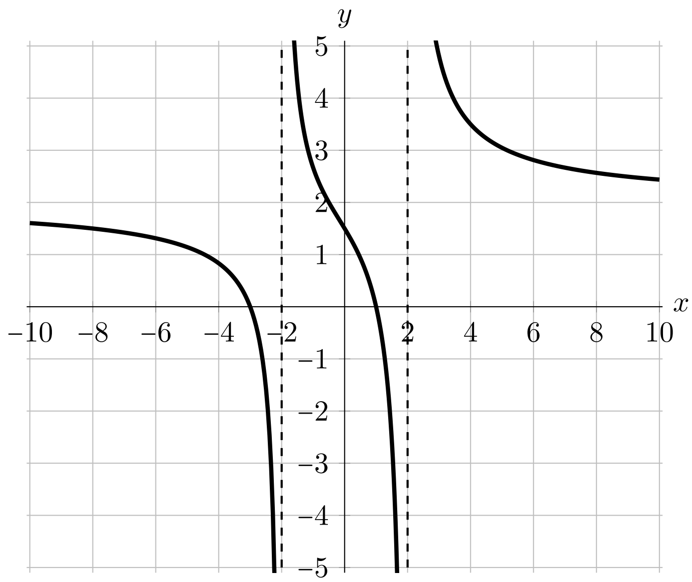
- $f(x)$ has zeros at $x = -1$, 2, and 4
- $\lim_{x \to -\infty} f(x) = -\infty$
- the y-intercept is 1
- $f(x)$ is of degree 4



Solution: There are two basic shapes that are possible here. For it to have the limit listed, a zero at $x = -1$, and vertical intercept at $y = 1$, then it must have a single zero at $x = -1$. However, for it to be degree four, it must have a multiplicity-2 zero at either $x = 2$ or $x = 4$. The graph shown above shows a multiplicity-2 zero at $x = 2$, but the other option is also possible.

b. [4 points] Write a possible formula for the graph of the rational function shown below. For clarity, its features are also described below.

- the y-intercept is 1.5
- the zeros are −3 and 1.
- horizontal asymptote of $y = 2$
- vertical asymptotes of $x = -2$ and $x = 2$



$y = \frac{2(x+3)(x-1)}{(x+2)(x-2)}$

Solution:
To begin with, since we have zerps of −3 and 1, we know we have factors of $(x+3)$ and $(x-1)$ in the numerator. Since we have vertical asymptotes at $x = -2$ and $x = 2$, we know we have factors of $(x+2)$ and $(x-2)$ in the denominator. As a preliminary function, we have so far:

$$\frac{(x+3)(x-1)}{(x+2)(x-2)}$$

However, we still need to account for the vertical intercept and horizontal asymptotes. We already have "matching degrees" in numerator and denominator, which will give us a non-zero horizontal asympote as desired. However, we want a horizontal asympote of $y = 2$, so we need the ratios of the leading coefficients to of numerator and denominator to be 2. We can edit our draft function above to get this:

$$\frac{2(x+3)(x-1)}{(x+2)(x-2)}$$

Finally, we should check that we have the right vertical intercept:

$$\frac{2(0+3)(0-1)}{(0+2)(0-2)} = \frac{-6}{-4} = 1.5$$

So we have met the last criteria as well!