9. [8 points]

a. [5 points] Find the values of the following limits. Your answer may be a numerical value, ∞, or $-\infty$. *You do not need to show work, but limited partial credit may be earned from work shown.*

(i) $\displaystyle \lim_{x\to 2} \frac{3(x-1)(x-2)}{(x-2)(x+3)} =$ ____________

Solution: This problem was removed from the exam.

(ii) $\displaystyle \lim_{x\to \infty} \frac{3(x-1)(x-2)}{(x-2)(x+3)} =$ **3**

Solution: Because both the numerator and denominator have degree 2, we need to look at the ratio of their leading coeffients to see what the value of the horizontal asymptote is. In this case, the leading coeffient of the numerator is 3 and the leading coefficient of the denonominator is 1, so the limit will approach $3/1 = 3$.

(iii) $\displaystyle \lim_{x\to \infty} \frac{x^8 - 7^x}{6^x + x^9} =$ $-\infty$

Solution: Because exponentially growing functions eventually dominate any polynomial, we need to only focus on 7^x and 6^x to determine the long-range behavior of this function. That is, the limit of the function given is the same as

$$\lim_{x\to\infty} \frac{-7^x}{6^x} = \lim_{x\to\infty} -\left(\frac{7}{6}\right)^x = -\infty$$

(iv) $\displaystyle \lim_{x\to\infty} \ln(x) =$ ∞

Solution: Even though the graph of $ln(x)$ seems to flatten out, it actually will grow without bound, to any arbitrarily large number. To reach an output of B, we need only input e^B, because $\ln(e^B) = B$. That is all to say, the limit in question is ∞.

b. [3 points] The weight w of a round melon is proportional to the cube of its radius r. That is,

$$w = kr^3,$$

where k is a constant. Currently, the melon's radius is 8 cm, and it weighs 5 pounds. How much would it weigh if its radius were to grow to 12 cm? *Give your answer in exact form or rounded to at least two decimals.*

Solution: We can use the information given to solve for the proportionality constant k:

$$5 = k8^3$$
$$k = \frac{5}{8^3}$$

Now we can use that value of k to find the weight w when $k = 12$ cm.

$$w = \frac{5}{8^3} \cdot 12^3 = 16.875$$

Answer: 16.875 pounds