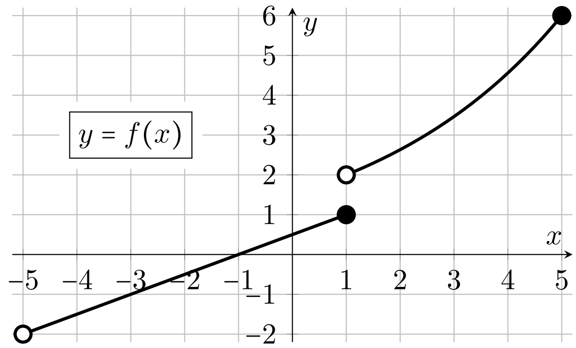
1. [14 points]

The entire graph of a function $f(x)$ is shown to the right. Also shown is a table of some values for two functions $g(x)$ and $h(x)$.

x	0	1	2	4
$g(x)$	2	−1	5	4
$h(x)$	0	0.5	0	8

The function $g(x)$ is defined for all real numbers and is **periodic** with a period of 5.



a. [3 points] Find the value of each of the following; write N/A if a value does not exist or there is not enough information to find it. *Showing work is not required, but may be eligible for partial credit in some cases.*

(i) $h(f(1)) =$ $h(1) = 0.5$

(ii) $g(\sin(20\pi)) =$ $g(0) = 2$

(iii) $g(f^{-1}(-1)) =$ $f(-3) = f(2) = 5$

b. [3 points] Suppose that we know further that $h(x)$ is a polynomial of **degree 3** with a double zero at $x = 2$. Combining this new knowledge with what's given in the table, find a formula for $h(x)$. *Show all work.*

Solution: Given that information about the zeros, we know that $h(x)$ must be of the form:

$$h(x) = ax(x-2)^2$$

for some constant a. To find the value of a we can plug in some known input/output combination and solve.

$$8 = a(4)(4-2)^2$$
$$8 = a(16)$$
$$\frac{1}{2} = a$$

$h(x) =$ $\frac{1}{2}x(x-2)^2$

c. [6 points] The piecewise function $f(x)$ consists of a linear piece and an exponential piece. Write a piecewise formula for the function $f(x)$. *Show all needed work.*

Solution: The linear part of the graph (on the left) has a domain of $-5 < x \leq 1$. We can see that the line goes up 1 for every 2 to the right, so has a slope of $\frac{1}{2}$. Finally, we can use point slope form and the fact that the line goes through $(1, 1)$ to find the formula for the linear part: $y = \frac{1}{2}(x-1) + 1$.

For the exponential part, we can notice first that the domain is $1 < x \leq 5$. There are a few ways to approach finding the formula. One way is to notice right away that the output multiplies by 3 (from 2 to 6) over 4 steps of x, meaning that $b^4 = 3$ so the growth factor is $b = 3^{\frac{1}{4}}$. If we think about that exponential part as a shift, then we can find a shortcut and think of it as $2(3^{\frac{1}{4}})^x$ shift one to the right, resulting in the formula: $2(3^{\frac{1}{4}})^{x-1}$.
We could also solve this by setting up a system of equations and unknown parameters (a and b) and use two known points.

$$2 = ab^1$$

$$6 = ab^5$$

Approaching it this way leads to an equivalent result:

$$\frac{2}{3^{\frac{1}{4}}}(3^{\frac{1}{4}})^t$$

$$f(x) = \begin{cases} \underline{\frac{1}{2}(x-1)+1} & \text{for } \underline{-5 < x \leq 1} \\ \underline{2(3^{\frac{1}{4}})^{x-1}} & \text{for } \underline{1 < x \leq 5} \end{cases}$$

d. [2 points] Find the domain of the function $f^{-1}(y)$ (*not* $f(x)$).

Domain of $f^{-1}(y)$: $\underline{(-2, 1] \cup (2, 6]}$

Solution: The domain of f^{-1} is the same as the *range* of f.