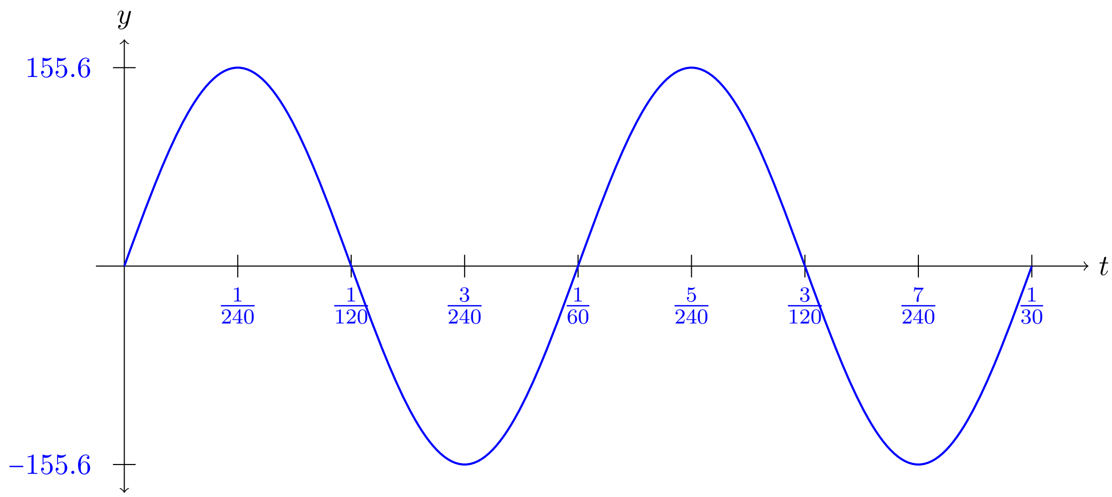
8. [11 points] In US households, electrical voltage (in volts) can be modeled by the function

$$V(t) = 155.6\sin(120\pi t)$$

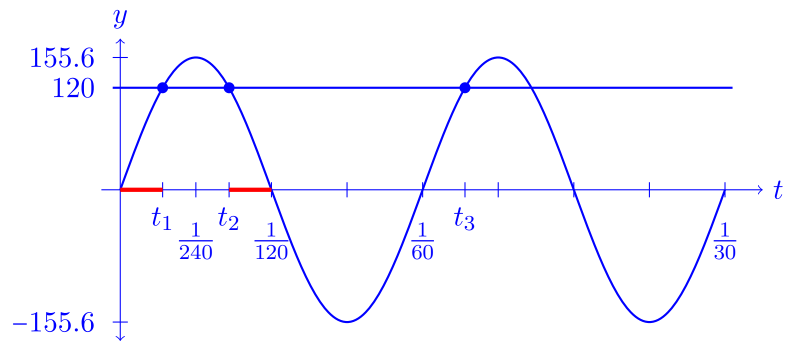
where t is measured in seconds.

a. [4 points] On the axes below, sketch a graph of **two periods** of $y = V(t)$. Your second cycle should end at exactly the furthest right tick on the t-axis. Clearly label at least two ticks on the t-axis. Use the ticks on the y-axis for your maximum and minimum values of $V(t)$ and label them as well.



b. [4 points] Find the first three positive values of t where the voltage is equal to 120 volts. *Show all work. Leave your answers in exact form or round to at least four decimal places.*

Solution: We want to find the first three positive values of t such that $V(t) = 155.6\sin(120\pi t) = 120$. We can mark the locations of these three solutions t_1, t_2, and t_3 on the graph we drew in part (a). We'll also label a few more tick marks to help us find the values of t_1, t_2, and t_3.



We first solve the equation $V(t) = 120$ for t algebraically:

$$\begin{aligned} 155.6\sin(120\pi t) &= 120 \\ \sin(120\pi t) &= \frac{120}{155.6} \\ 120\pi t &= \arcsin(120/155.6) \\ t &= \frac{\arcsin(120/155.6)}{120\pi} \approx 0.0023 \end{aligned}$$

We can check that this value of t is between 0 and $\frac{1}{240} \approx 0.0042$, so this must be the value of t_1. The third solution t_3 is then equal to $t_1 + 1/60 = \frac{\arcsin(120/155.6)}{120\pi} + \frac{1}{60} \approx 0.0190$ since the period of the function is $1/60$. To find the second solution, we note that the distance from 0 to t_1 is the same as the distance from t_2 to $\frac{1}{120}$. These two distances are shown in red in the graph above. This tells us that $t_1 - 0 = \frac{1}{120} - t_2$, so $t_2 = \frac{1}{120} - t_1 = \frac{1}{120} - \frac{\arcsin(120/155.6)}{120\pi} \approx 0.0060$

$t =$ $\frac{\arcsin(120/155.6)}{120\pi} \approx 0.0023$, $\frac{1}{120} - \frac{\arcsin(120/155.6)}{120\pi} \approx 0.0060$, $\frac{\arcsin(120/155.6)}{120\pi} + \frac{1}{60} \approx 0.0190$

c. [3 points] In Australia, the voltage alternates between a maximum of 240 volts, to a minimum of −240 volts, and back to 240 volts 50 times per second. Find a formula for the function $A(t)$ which models the voltage in Australia t seconds from when the voltage is at its **maximum**.

Solution: Since the voltage oscillates back and forth and starts at a maximum when $t = 0$, we will choose to express $A(t)$ as a transformation of $\cos(t)$. The amplitude must be 240, and the period must be $1/50$. Therefore $A(t) = 240\cos(100\pi t)$.

$A(t) =$ $240\cos(100\pi t)$