9. [7 points] Jay drinks a cup of coffee which contains 140mg of caffeine. The amount of caffeine, in milligrams, left in his body h hours after he drinks the coffee is given by the function

$$C(h) = 140(0.89)^h.$$

For each part below, show all work and leave your answer in exact form or rounded to at least two decimal places.

a. [2 points] What percent of the caffeine in the cup of coffee is left in Jay's body 2 hours after he drinks it?

Solution: After 2 hours, Jay has $C(2) = 140(0.89)^2$ mg of caffeine left in his body. This is $100 \cdot (0.89)^2\% = 79.21\%$ of the 140mg of caffeine that was in the cup of coffee.

79.21 %.

b. [2 points] What is the continuous percent decay rate of $C(h)$?

Solution: The growth factor of the exponential function $C(h)$ is 0.89. Therefore, the continuous decay rate as a decimal is $\ln(0.89) \approx -0.1165$. Therefore, the continuous percent decay rate is $100 \cdot (-\ln(0.89))\%$, or 11.65%.

$100 \cdot (-\ln(0.89)) \approx 11.65$ %

c. [3 points] Find the half-life of $C(h)$. *Include relevant units in your answer.*

Solution: We want to find the value of h such that $C(h) = 0.5C(0)$. Solving this equation for h gives us

$$\begin{aligned} C(h) &= 0.5C(0) \\ 140(0.89)^h &= 0.5(140)(0.89)^0 \\ 140(0.89)^h &= 0.5(140) \\ 0.89^h &= 0.5 \\ \log(0.89^h) &= \log(0.5) \\ h\log(0.89) &= \log(0.5) \\ h &= \frac{\log(0.5)}{\log(0.89)} \end{aligned}$$

The half-life of $C(h)$ is $\frac{\log(0.5)}{\log(0.89)} \approx 5.95$ **hours**.