2. [11 points] *For full credit on this problem, you must show your work carefully. Unless specified otherwise, answers should either be in exact form or be rounded accurately to at least three decimal places.*

The population of Linearville grew from 9,000 in January 2003 to 14,000 in January 2005.

a. [2 points] Find the average rate of change of the population of Linearville between January 2003 and January 2005. (*Include units.*)

Solution: Since the population grew by 5,000 people in the two years from January 2003 to January 2005, the average rate of change is $\frac{5000}{2} = 2500$ people per year.

Answer: 2500 **people per year**

b. [3 points] The population of Linearville has been growing linearly since January 2000. Find a formula for $L(t)$, the population of Linearville t years after January 2000.

Solution: The average rate of change from part (a) gives the slope of $L(t)$. Since the population was 9,000 in January 2003, we see that $L(3) = 9000$. Using point-slope form, we find that $L(t) = 9000 + 2500(t - 3)$ or $L(t) = 1500 + 2500t$.

Answer: $L(t) = 1500 + 2500t$

The neighboring town of Exponential Corner has also been growing. Its population was 100 in January 2003 and had risen to 150 by January 2005.

c. [4 points] Suppose that since January 2000, the population of Exponential Corner has been growing exponentially. Find a formula for $E(t)$, the population of Exponential Corner t years after January 2000.

Solution: Since the function is exponential, a formula for $E(t)$ is given by $E(t) = ab^t$ for some constants a and b. We are told that $E(3) = 100$ and $E(5) = 150$, so we find that $100 = ab^3$ and $150 = ab^5$. Dividing, we see that $\frac{150}{100} = \frac{ab^5}{ab^3}$ so $1.5 = b^2$. Then $b = \sqrt{1.5} \approx 1.225$ and we can solve for a. In particular $100 = a(\sqrt{1.5})^3$ so $a = \frac{100}{\sqrt{1.5}^3} \approx 54.433$. Thus $E(t) = \frac{100}{(\sqrt{1.5})^3}(\sqrt{1.5})^t \approx 54.433(1.225)^t$.

Answer: $E(t) = \frac{100}{(\sqrt{1.5})^3}(\sqrt{1.5})^t \approx 54.433(1.225)^t$

d. [2 points] Assuming the populations continue to grow as described above, will the population of Exponential Corner ever catch up to the population of Linearville?
If so, when will this happen? (*Round to the nearest year.*)
If not, explain how you know this.

Solution: Since $E(t)$ is an increasing positive exponential function, it will eventually dominate the linear function $L(t)$. Using a graphing calculator's "intersect" feature, we find that $L(t) = E(t)$ when $t \approx 36.7$. So, the population of Exponential Corner will catch up to that of Linearville in late 2036 at which time both populations will be about 93,332.