3. [15 points] Show your work.
   
   a. [3 points] Find an equation for the straight line passing through the point (2, −3) that is perpendicular to the line passing through the points (1, 4) and (−6, 5).
      
      Solution: The slope of the line passing through (1, 4) and (−6, 5) is \( \frac{5-4}{-6-1} = -\frac{1}{7} \), so the slope of a line perpendicular to it is 7. Using point-slope form, we therefore find that \( y + 3 = 7(x - 2) \) describes the perpendicular line passing through the point (2, −3).
      
      Answer: \( y = 7(x - 2) - 3 \) or \( -17 + 7x \)
   
   b. [3 points] A population of ants is growing by 25% per day. How long will it take for the number of ants to double? (Give your answer in exact form or rounded accurately to three decimal places.)
      
      Solution: The population can be modeled by an exponential function of the form \( a(1.25)^t \) where \( t \) is measured in days and \( a \) is the population on day \( t = 0 \). If \( d \) is the doubling time, then \( 2a = a(1.25)^d \) so \( 2 = 1.25^d \). Solving for \( d \) we have \( \ln 2 = d \ln 1.25 \) so \( d = \frac{\ln 2}{\ln 1.25} \approx 3.106 \).
      
      Answer: \( d = \frac{\ln 2}{\ln 1.25} \approx 3.106 \) days
   
   c. [3 points] An ant begins at the point (1, 0) and walks counterclockwise along the unit circle for a distance of 2 units and then stops. What are the coordinates of the point at which the ant stops? (Give your answer in exact form.)
      
      Solution: On the unit circle, a distance of 2 units corresponds to an angle of 2 radians, so this counterclockwise walk starts at (1, 0) and ends at the point \( (\cos 2, \sin 2) \).
      
      Answer: \( (\cos 2, \sin 2) \)
   
   d. [3 points] Suppose the graph of \( y = h(x) \) is obtained from the graph of \( y = 3e^{2x} \) by shifting the graph of \( y = 3e^{2x} \) to the right four units and then down five units. Find a formula for \( h(x) \).
      
      Solution: Let \( f(x) = 3e^{2x} \). Then \( h(x) = f(x - 4) - 5 = 3e^{2(x-4)} - 5 \).
      
      Answer: \( h(x) = 3e^{2(x-4)} - 5 \)
   
   e. [3 points] Find the exact value of \( t \) if \( 5e^t = 15(2^t) \). (Show each step of your work carefully.)
      
      Solution: First, we divide both sides of the given equation by 5 to obtain \( e^t = 3(2^t) \). Using logarithms, we then find \( \ln(e^t) = \ln(3(2^t)) \) so \( t = \ln(3) + \ln(2^t) = \ln 3 + t \ln 2 \). Collecting the terms involving \( t \), we find \( t - t \ln 2 = \ln 3 \). We can then pull out the common factor \( t \) on the left side to find \( t(1 - \ln 2) = \ln 3 \) and then divide to obtain the solution \( t = \frac{\ln 3}{1 - \ln 2} \).
      
      Answer: \( t = \frac{\ln 3}{1 - \ln 2} \)