3. [15 points] *Show your work.*

a. [3 points] Find an equation for the straight line passing through the point $(2, -3)$ that is perpendicular to the line passing through the points $(1, 4)$ and $(-6, 5)$.

Solution: The slope of the line passing through $(1, 4)$ and $(-6, 5)$ is $\frac{5-4}{-6-1} = -\frac{1}{7}$, so the slope of a line perpendicular to it is 7. Using point-slope form, we therefore find that $y + 3 = 7(x - 2)$ describes the perpendicular line passing through the point $(2, -3)$.

Answer: $y =$ $7(x-2) - 3$ **or** $-17 + 7x$

b. [3 points] A population of ants is growing by 25% per day. How long will it take for the number of ants to double? (*Give your answer in exact form or rounded accurately to three decimal places.*)

Solution: The population can be modeled by an exponential function of the form $a(1.25)^t$ where t is measured in days and a is the population on day $t = 0$. If d is the doubling time, then $2a = a(1.25)^d$ so $2 = 1.25^d$. Solving for d we have $\ln 2 = d \ln 1.25$ so $d = \dfrac{\ln 2}{\ln 1.25} \approx 3.106$.

Answer: $d = \dfrac{\ln 2}{\ln 1.25} \approx 3.106$ **days**

c. [3 points] An ant begins at the point $(1, 0)$ and walks *counterclockwise* along the unit circle for a distance of 2 units and then stops. What are the coordinates of the point at which the ant stops? (*Give your answer in exact form.*)

Solution: On the unit circle, a distance of 2 units corresponds to an angle of 2 radians, so this counterclockwise walk starts at $(1, 0)$ and ends at the point $(\cos 2, \sin 2)$.

Answer: $(\cos 2, \sin 2)$

d. [3 points] Suppose the graph of $y = h(x)$ is obtained from the graph of $y = 3e^{2x}$ by shifting the graph of $y = 3e^{2x}$ to the right four units and then down five units. Find a formula for $h(x)$.

Solution: Let $f(x) = 3e^{2x}$. Then $h(x) = f(x - 4) - 5 = 3e^{2(x-4)} - 5$.

Answer: $h(x) =$ $3e^{2(x-4)} - 5$

e. [3 points] Find the *exact* value of t if $5e^t = 15(2^t)$. (*Show each step of your work carefully.*)

Solution: First, we divide both sides of the given equation by 5 to obtain $e^t = 3(2^t)$. Using logarithms, we then find $\ln(e^t) = \ln(3(2^t))$ so $t = \ln(3) + \ln(2^t) = \ln 3 + t \ln 2$. Collecting the terms involving t, we find $t - t \ln 2 = \ln 3$. We can then pull out the common factor t on the left side to find $t(1 - \ln 2) = \ln 3$ and then divide to obtain the solution $t = \dfrac{\ln 3}{1 - \ln 2}$.

Answer: $t = \dfrac{\ln 3}{1 - \ln 2}$