5. [6 points] Let g be the function defined by

$$g(x) = \frac{10(x-1)(x-2)}{(2x+1)(x^2+2x-1)}.$$

Find all zeros, y-intercepts, and horizontal and vertical asymptotes of the graph of $y = g(x)$. If appropriate, write "NONE" in the answer blank provided.
(*Show your work and write your answers in exact form.*)

Solution: Note that the numerator and denominator have no common factors.

The zeros of $g(x)$ are the zeros of $10(x-1)(x-2)$ which are $x = 1$ and $x = 2$.

zero(s): $x = 1$ **and** $x = 2$

The y-intercept of $g(x)$ is $g(0) = \dfrac{10(0-1)(0-2)}{(2(0)+1)(0^2+2(0)-1)} = \dfrac{20}{-1} = -20$.

y-intercept(s): -20

As $x \to \pm\infty$ the numerator and denominator of $g(x)$ behave like their leading terms, so as $x \to \pm\infty$, the rational function $g(x)$ behaves like $\frac{10x^2}{2x^3} = \frac{5}{x}$ which approaches 0 as $x \to \pm\infty$. Therefore, $y = 0$ is a horizontal asymptote of the graph of $y = g(x)$.

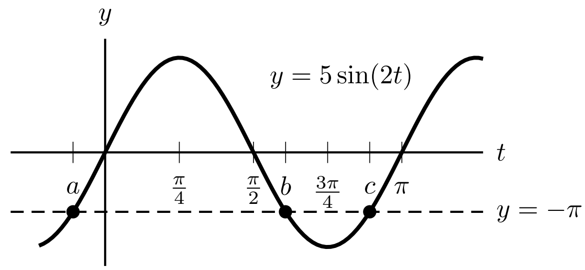
horizontal asymptote(s): $y = 0$

The vertical asymptotes of $g(x)$ are determined by the zeros of the denominator, i.e. the solutions to $(2x+1)(x^2+2x-1) = 0$ which are the solution of $2x+1 = 0$ along with the solutions of $x^2+2x+1 = 0$. The solution of $2x+1 = 0$ is $x = -1/2$ and by applying the quadratic formula, we see that the solutions of $x^2+2x-1 = 0$ are $x = \frac{-2\pm\sqrt{2^2-4(1)(-1)}}{2(1)} = \frac{-2\pm\sqrt{8}}{2} = -1 \pm \sqrt{2}$. So the graph of $y = g(x)$ has three vertical asymptotes: $x = -1/2$, $x = -1+\sqrt{2}$ and $x = -1-\sqrt{2}$.

vertical asymptote(s): $x = -1/2$, $x = -1+\sqrt{2}$, **and** $x = -1-\sqrt{2}$

6. [5 points] Find all solutions to the equation $5\sin(2t) = -\pi$ for $0 \le t \le \pi$.
(*Show your work clearly and give your final answer(s) in exact form.*)

Solution:
We first find one solution to the given equation. Since $5\sin(2t) = -\pi$ we have $\sin(2t) = -\pi/5$. Then one solution is given by $2t = \arcsin(-\pi/5)$, i.e. $t = \frac{1}{2}\arcsin\left(-\frac{\pi}{5}\right)$. To find all solutions in the domain $[0, \pi]$, we consider the graph of $y = 5\sin(2t)$ (shown to the right). The graph intersects the line $y = -\pi$ at two points for $0 \le t \le \pi$. These two points of intersection give the solutions b and c marked on the t–axis. These are the solutions we are looking for. The solution we found algebraically is marked a on the t-axis.
From the graph, we see that c is exactly one period after a, that is $c = \pi + a$.
Using symmetry, we find that $b = \frac{\pi}{2} - a$. (There are many ways to see this. For example, note that $\frac{3\pi}{4}$ is halfway between b and c. Therefore $\frac{b+c}{2} = \frac{3\pi}{4}$ so $b = \frac{3\pi}{2} - c = \frac{3\pi}{2} - (\pi + a) = \frac{\pi}{2} - a$.) Hence the two solutions to the equation $5\sin(2t) = -\pi$ for $0 \le t \le \pi$ are $t = \pi + \frac{1}{2}\arcsin\left(-\frac{\pi}{5}\right)$ and $t = \frac{\pi}{2} - \frac{1}{2}\arcsin\left(-\frac{\pi}{5}\right)$.



Answer(s): $t = \pi + \dfrac{1}{2}\arcsin\left(-\dfrac{\pi}{5}\right)$ **and** $t = \dfrac{\pi}{2} - \dfrac{1}{2}\arcsin\left(-\dfrac{\pi}{5}\right)$