

5. [6 points] Let  $g$  be the function defined by

$$g(x) = \frac{10(x-1)(x-2)}{(2x+1)(x^2+2x-1)}.$$

Find all zeros,  $y$ -intercepts, and horizontal and vertical asymptotes of the graph of  $y = g(x)$ . If appropriate, write “NONE” in the answer blank provided. (Show your work and write your answers in exact form.)

**Solution:** Note that the numerator and denominator have no common factors.

The zeros of  $g(x)$  are the zeros of  $10(x-1)(x-2)$  which are  $x = 1$  and  $x = 2$ .

$$\text{zero(s): } \underline{x = 1 \text{ and } x = 2}$$

The  $y$ -intercept of  $g(x)$  is  $g(0) = \frac{10(0-1)(0-2)}{(2(0)+1)(0^2+2(0)-1)} = \frac{20}{-1} = -20$ .

$$\text{y-intercept(s): } \underline{-20}$$

As  $x \rightarrow \pm\infty$  the numerator and denominator of  $g(x)$  behave like their leading terms, so as  $x \rightarrow \pm\infty$ , the rational function  $g(x)$  behaves like  $\frac{10x^2}{2x^3} = \frac{5}{x}$  which approaches 0 as  $x \rightarrow \pm\infty$ . Therefore,  $y = 0$  is a horizontal asymptote of the graph of  $y = g(x)$ .

$$\text{horizontal asymptote(s): } \underline{y = 0}$$

The vertical asymptotes of  $g(x)$  are determined by the zeros of the denominator, i.e. the solutions to  $(2x+1)(x^2+2x-1) = 0$  which are the solution of  $2x+1 = 0$  along with the solutions of  $x^2+2x-1 = 0$ . The solution of  $2x+1 = 0$  is  $x = -1/2$  and by applying the quadratic formula, we see that the solutions of  $x^2 + 2x - 1 = 0$  are  $x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)} = \frac{-2 \pm \sqrt{8}}{2} = -1 \pm \sqrt{2}$ . So the graph of  $y = g(x)$  has three vertical asymptotes:  $x = -1/2$ ,  $x = -1 + \sqrt{2}$  and  $x = -1 - \sqrt{2}$ .

$$\text{vertical asymptote(s): } \underline{x = -1/2, x = -1 + \sqrt{2}, \text{ and } x = -1 - \sqrt{2}}$$

6. [5 points] Find all solutions to the equation  $5 \sin(2t) = -\pi$  for  $0 \leq t \leq \pi$ . (Show your work clearly and give your final answer(s) in exact form.)

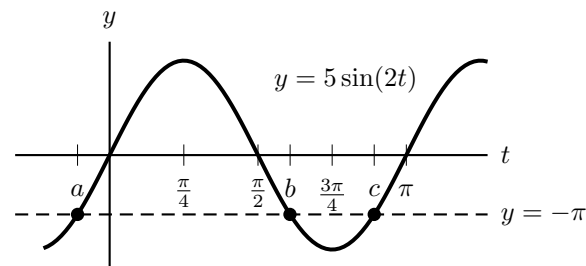
**Solution:**

We first find one solution to the given equation. Since  $5 \sin(2t) = -\pi$  we have  $\sin(2t) = -\pi/5$ . Then one solution is given by  $2t = \arcsin(-\pi/5)$ , i.e.  $t = \frac{1}{2} \arcsin(-\pi/5)$ . To find all solutions in the domain  $[0, \pi]$ , we consider the graph of  $y = 5 \sin(2t)$  (shown to the right). The graph intersects the line  $y = -\pi$  at two points for  $0 \leq t \leq \pi$ . These two points of intersection give the solutions  $b$  and  $c$  marked on the  $t$ -axis. These are the solutions we are looking for. The solution we found algebraically is marked  $a$  on the  $t$ -axis.

From the graph, we see that  $c$  is exactly one period after  $a$ , that is  $c = \pi + a$ .

Using symmetry, we find that  $b = \frac{\pi}{2} - a$ . (There are many ways to see this. For example, note that

is halfway between  $b$  and  $c$ . Therefore  $\frac{b+c}{2} = \frac{3\pi}{4}$  so  $b = \frac{3\pi}{2} - c = \frac{3\pi}{2} - (\pi + a) = \frac{\pi}{2} - a$ .) Hence the two solutions to the equation  $5 \sin(2t) = -\pi$  for  $0 \leq t \leq \pi$  are  $t = \pi + \frac{1}{2} \arcsin(-\pi/5)$  and  $t = \frac{\pi}{2} - \frac{1}{2} \arcsin(-\pi/5)$ .



$$t = \pi + \frac{1}{2} \arcsin\left(-\frac{\pi}{5}\right) \text{ and } t = \frac{\pi}{2} - \frac{1}{2} \arcsin\left(-\frac{\pi}{5}\right)$$

$$\text{Answer(s): } \underline{\hspace{10em}}$$