7. [11 points] In honor of a favorite video game, a group of students decides to build a huge slingshot on the Diag from which they will launch a variety of large toy stuffed animals.

The first “passenger” is a large stuffed panda. The height of the panda above the ground (measured in feet) \( t \) seconds after it is launched from the slingshot is \( P(t) = -16t^2 + 48t + 8 \).

a. [3 points] How long is the flying stuffed panda in the air before it lands back on the ground? (Show your work and give your answer in exact form or rounded to three decimal places.)

**Solution:** The panda lands on the ground when \( P(t) = 0 \), i.e. when \(-16t^2 + 48t + 8 = 0\). We first divide both sides of this equation by \(-8\) to simplify it to \(2t^2 - 6t - 1 = 0\).

Applying the quadratic formula* we find \( t = \frac{3 \pm \sqrt{11}}{2} \).

The solution that makes sense in context is the positive one, i.e. \( t = \frac{3 + \sqrt{11}}{2} \approx 3.158 \). Hence, the flying stuffed panda is in the air for approximately 3.158 seconds before it lands on the ground.

*(Note that we could instead use a graphing calculator to estimate the positive zero of the function \( P(t) \).)

**Answer:** \( t = \frac{3 + \sqrt{11}}{2} \approx 3.158 \text{ seconds} \)

b. [4 points] Use the method of completing the square to rewrite the formula for \( P(t) \) in vertex form. (Carefully show your work step-by-step.)

**Solution:**

\[
P(t) = -16t^2 + 48t + 8 = -16(t^2 - 3t) + 8 \quad \text{(factor out leading coefficient)}
\]

\[
= -16 \left( t^2 - 3t + \frac{9}{4} - \frac{9}{4} \right) + 8 \quad \text{(add and subtract \(- \frac{3}{2}\)^2 = \frac{9}{4})}
\]

\[
= -16 \left( t - \frac{3}{2} \right)^2 - 16 \left( - \frac{3}{2} \right)^2 + 8 \quad \text{(rewrite perfect square)}
\]

\[
= -16 \left( t - \frac{3}{2} \right)^2 + 36 + 8 = -16 \left( t - \frac{3}{2} \right)^2 + 44 \quad \text{(distribute and simplify)}
\]

**Answer:** \( P(t) = -16 \left( t - \frac{3}{2} \right)^2 + 44 \)

c. [2 points] After how many seconds does the flying stuffed panda reach its maximum height above the ground? What is that maximum height?

**Solution:** We see from part (b) that the vertex of \( y = P(t) \) is the point \( (1.5, 44) \). Since the leading coefficient if \( P(t) \) is negative, the graph of \( P \) is a parabola that opens downward, so \( P \) achieves a maximum at this vertex.

After \( 1.5 \) seconds, the panda reaches its maximum height of \( 44 \) feet.

d. [2 points] In the context of this problem, what are the domain and range of \( P(t) \)? (Use either inequalities or interval notation to give your answers.)

**Solution:** The answers to parts (a) and (c) above give us the domain and range.

**Domain:** \( [0, (3 + \sqrt{11})/2] \) \hspace{1cm} **Range:** \( [0, 44] \)