7. [11 points] In honor of a favorite video game, a group of students decides to build a huge slingshot on the Diag from which they will launch a variety of large toy stuffed animals.

The first "passenger" is a large stuffed panda. The height of the panda above the ground (measured in feet) t seconds after it is launched from the slingshot is $P(t) = -16t^2 + 48t + 8$.

a. [3 points] How long is the flying stuffed panda in the air before it lands back on the ground? (*Show your work and give your answer in exact form or rounded to three decimal places.*)

Solution: The panda lands on the ground when $P(t) = 0$, i.e. when $-16t^2 + 48t + 8 = 0$. We first divide both sides of this equation by -8 to simplify it to $2t^2 - 6t - 1 = 0$.
Applying the quadratic formula* we find $t = \dfrac{6 \pm \sqrt{(-6)^2 - 4(2)(-1)}}{2(2)} = \dfrac{6 \pm \sqrt{44}}{4} = \dfrac{3 \pm \sqrt{11}}{2}$.
The solution that makes sense in context is the positive one, i.e. $t = \dfrac{3 + \sqrt{11}}{2} \approx 3.158$. Hence, the flying stuffed panda is in the air for approximately 3.158 seconds before it lands on the ground.
(*Note that we could instead use a graphing calculator to estimate the positive zero of the function P.)

Answer: $\dfrac{3+\sqrt{11}}{2} \approx 3.158$ **seconds**

b. [4 points] Use the method of completing the square to rewrite the formula for $P(t)$ in vertex form. (*Carefully show your work step-by-step.*)

Solution:

$$\begin{aligned} P(t) &= -16t^2 + 48t + 8 = -16(t^2 - 3t) + 8 \quad \text{(factor out leading coefficient)} \\ &= -16\left(t^2 - 3t + \frac{9}{4} - \frac{9}{4}\right) + 8 \quad \text{(add and subtract } \left(-\tfrac{3}{2}\right)^2 = \tfrac{9}{4}\text{)} \\ &= -16\left(\left(t - \frac{3}{2}\right)^2 - \frac{9}{4}\right) + 8 \quad \text{(rewrite perfect square)} \\ &= -16\left(t - \frac{3}{2}\right)^2 + 36 + 8 = -16\left(t - \frac{3}{2}\right)^2 + 44 \quad \text{(distribute and simplify)} \end{aligned}$$

Answer: $P(t) = -16\left(t - \dfrac{3}{2}\right)^2 + 44$

c. [2 points] After how many seconds does the flying stuffed panda reach its maximum height above the ground? What is that maximum height?

Solution: We see from part (b) that the vertex of $y = P(t)$ is the point $(1.5, 44)$. Since the leading coefficient if $P(t)$ is negative, the graph of P is a parabola that opens downward, so P achieves a maximum at this vertex.

After 1.5 seconds, the panda reaches its maximum height of 44 feet.

d. [2 points] In the context of this problem, what are the domain and range of $P(t)$? (*Use either inequalities or interval notation to give your answers.*)

Solution: The answers to parts (a) and (c) above give us the domain and range.

Domain: $[0, (3 + \sqrt{11})/2]$ **Range:** $[0, 44]$