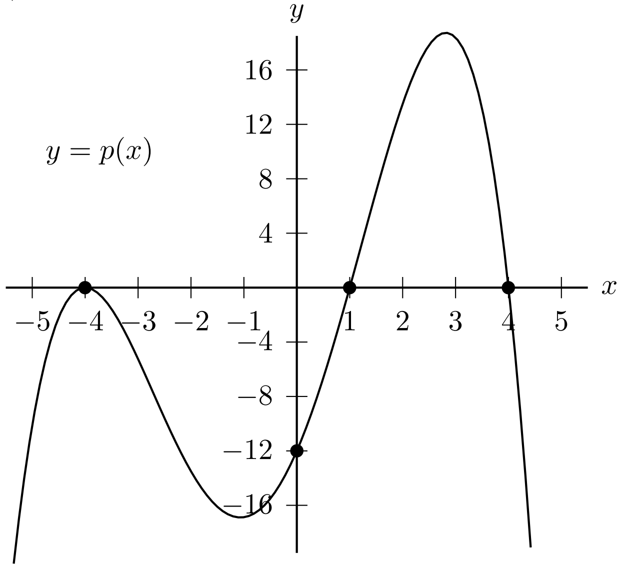
8. [5 points] A portion of the graph of a polynomial function p is shown below.
Find a possible formula for $p(x)$.
(*Assume all of the key features of the graph are shown.*)



Solution: The zeros of $p(x)$ are $x = -4$, $x = 1$, and $x = 4$. Note that $x = -4$ is a double (or other positive even power) zero while $x = 1$ and $x = 4$ appear to be simple zeros. So a possible formula for $p(x)$ is $p(x) = a(x+4)^2(x-1)(x-4)$ for some (negative) constant a. Using the y-intercept, we see that $-12 = a(0+4)^2(0-1)(0-4)$, so $a = -12/64 = -3/16$.

Answer: $p(x) =$ $-\frac{3}{16}(x+4)^2(x-1)(x-4)$

9. [4 points] Suppose g is a power function such that $g(1) = 4$ and $g(10) = 1$.
Find a formula for $g(x)$. (*Any numbers in your formula should be in exact form.*)

Solution: Since g is a power function there are constants k and p so that a formula for $g(x)$ is $g(x) = kx^p$. Using the given data, we have $4 = k(1^p)$ so $4 = k$. Then $1 = k(10^p) = 4(10^p)$. Solving for p we have

$$\begin{aligned} 4(10^p) &= 1 \\ 10^p &= \frac{1}{4} \\ \log(10^p) &= \log(1/4) \\ p &= -\log 4 \end{aligned}$$

Hence a formula for $g(x)$ is $g(x) = 4x^{-\log 4}$ (which can also be written as $\frac{4}{x^{\log 4}}$ or $4x^{\log 0.25}$).

Check: $g(1) = 4(1^{-\log 4}) = 4(1) = 4$ and $g(10) = 4(10^{-\log 4}) = \frac{4}{10^{\log 4}} = \frac{4}{4} = 1$ as required.

Answer: $g(x) =$ $4x^{-\log 4}$