8. [5 points] A portion of the graph of a polynomial function $p$ is shown below. Find a possible formula for $p(x)$.

(Assume all of the key features of the graph are shown.)

\[y = p(x)\]

\[x\] \hspace{1cm} \[y\]

\[\begin{array}{c}
-5 & -4 & -3 & -2 & -1 & 1 & 2 & 3 & 4 & 5 \\
-16 & -12 & -8 & -4 & 0 & 4 & 8 & 12 & 16
\end{array}\]

**Solution:** The zeros of $p(x)$ are $x = -4$, $x = 1$, and $x = 4$. Note that $x = -4$ is a double (or other positive even power) zero while $x = 1$ and $x = 4$ appear to be simple zeros. So a possible formula for $p(x)$ is $p(x) = a(x + 4)^2(x - 1)(x - 4)$ for some (negative) constant $a$.

Using the $y$-intercept, we see that $-12 = a(0 + 4)^2(0 - 1)(0 - 4)$, so $a = -12/64 = -3/16$.

**Answer:** $p(x) = -\frac{3}{16}(x + 4)^2(x - 1)(x - 4)$

9. [4 points] Suppose $g$ is a power function such that $g(1) = 4$ and $g(10) = 1$. Find a formula for $g(x)$. (Any numbers in your formula should be in exact form.)

**Solution:** Since $g$ is a power function there are constants $k$ and $p$ so that a formula for $g(x)$ is $g(x) = kx^p$. Using the given data, we have $4 = k(1^p)$ so $4 = k$. Then $1 = k(10^p) = 4(10^p)$.

Solving for $p$ we have

\[
\begin{align*}
4(10^p) &= 1 \\
10^p &= \frac{1}{4} \\
\log(10^p) &= \log(\frac{1}{4}) \\
p &= -\log(4)
\end{align*}
\]

Hence a formula for $g(x)$ is $g(x) = 4x^{-\log 4}$ (which can also be written as $\frac{4}{x^{\log 4}}$ or $4x^{\log 0.25}$).

**Check:** $g(1) = 4(1^{-\log 4}) = 4(1) = 4$ and $g(10) = 4(10^{-\log 4}) = \frac{4}{10^{\log 4}} = \frac{4}{4} = 1$ as required.

**Answer:** $g(x) = 4x^{-\log 4}$