10. [8 points]

a. [4 points] Let $y = f(x) = 3\log\left(\dfrac{1+2x}{x+3}\right)$. Find a formula for $f^{-1}(y)$. Show all your work carefully.

Solution:

$$\begin{aligned} y &= 3\log\left(\frac{1+2x}{x+3}\right) \\ \frac{y}{3} &= \log\left(\frac{1+2x}{x+3}\right) \\ 10^{\frac{y}{3}} &= \frac{1+2x}{x+3} \\ 10^{\frac{y}{3}}(x+3) &= 1+2x \\ 10^{\frac{y}{3}}x + 3(10^{\frac{y}{3}}) &= 1+2x \\ (10^{\frac{y}{3}}-2)x &= 1-3(10^{\frac{y}{3}}) \\ f^{-1}(y) &= \frac{1-3(10^{\frac{y}{3}})}{10^{\frac{y}{3}}-2}. \end{aligned}$$

b. [4 points] Find all solutions to $3\cos\left(\dfrac{t}{2}\right)+2=0$ for $0 \le t \le 4\pi$ algebraically. Show all your work carefully. Your answer(s) must be in **exact form**.

Solution:

$$\begin{aligned} 3\cos\left(\frac{t}{2}\right)+2 &= 0 \\ \cos\left(\frac{t}{2}\right) &= -\frac{2}{3}. \\ \frac{t}{2} &= \cos^{-1}\left(-\frac{2}{3}\right). \end{aligned}$$

$$t_1 = 2\cos^{-1}\left(-\frac{2}{3}\right), \quad t_2 = 4\pi - 2\cos^{-1}\left(-\frac{2}{3}\right)$$