4. [12 points]

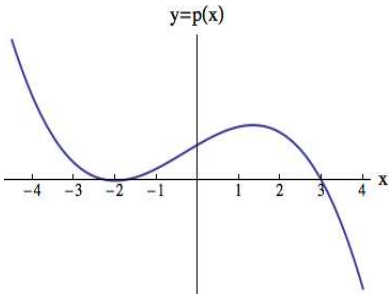
a. [4 points] The graph of a polynomial $p(x)$ is shown below. The following facts are known about $p(x)$:

i) The only zeros of $p(x)$ are $x = -2$ and $x = 3$.

ii) The degree of $p(x)$ is at most four.

iii) The point $(1, 9)$ is on the graph of $p(x)$.

Find a formula for $p(x)$.



Solution: The polynomial $p(x)$ has degree 3 given the long behavior in the graph. From the graph, we can see that $x = -2$ is a double zero. Hence $p(x) = k(x+2)^2(x-3)$. Since the point $(1, 9)$ is on the graph of $p(x)$, then $9 = k(3^2)(-2) = -18k$. Hence $k = -0.5$. Then $p(x) = -0.5(x + 2)^2(x - 3)$.

b. [5 points] Let

$$R(x) = \frac{(x^2 + 9)(10x + 1)}{7x^3 - x}.$$

Find all the intercepts and all horizontal and vertical asymptotes of the graph $y = R(x)$. If appropriate, write "None" in the answer blank provided. Your answers should be in **exact form**.

Solution:

i) x-intercept(s): Set $(x^2 + 9)(10x + 1) = 0$. Then $x^2 + 9 = 0$ (has no solutions) and $10x + 1 = 0$ implies $x = -0.1$.

ii) y-intercept(s): Since $R(0)$ is undefined, then $y = R(x)$ has **no y-intercepts**.

iii) vertical asymptote(s): Set $7x^3 - x = 0$, then $x(7x^2 - 1) = 0$ which yields $x = 0$ and $\mathbf{x} = \pm\frac{1}{\sqrt{7}}$

iv) horizontal asymptote(s): $y = \frac{10}{7}$.

c. [3 points] A law of physics states that the force F (in Newtons) exerted between two objects is inversely proportional to the square of the distance r (in meters) between them, and $F = 30$ when $r = 7$. Find a formula for F in terms of r.

Solution: Since F is inversely proportional to r^2, then $F(r) = \frac{k}{r^2}$. Using $F(7) = 30$, we get $k = (30)(7)^2 = 1,470$. Hence $F(r) = \frac{1470}{r^2}$.