5. [11 points] A package is thrown from an airplane. The height of the package (in meters) above the ground t seconds after it was thrown from the airplane is given by the function

$$H(t) = -5t^2 - 10t + 160.$$

a. [2 points] What is the height of the airplane at the time in which the package is thrown? Include units.

Height= ____________________

Solution: Height$=H(0) = 160$ meters.

b. [3 points] How many seconds does it take for the package to be 10 meters above the ground? Find your answer algebraically. Show all your work.

Solution: Solve $H(t) = 10$. In this case $-5t^2 - 10t + 160 = 10$, or $-5t^2 - 10t + 150 = 0$. Using the quadratic formula

$$t = \frac{10 \pm \sqrt{100 - 4(-5)(150)}}{-10} = \frac{10 \pm \sqrt{3100}}{-10} = -1 \pm \sqrt{31}.$$

It takes $-1 + \sqrt{31} \approx 4.56$ seconds for the package to be 10 meters above the ground.

c. [2 points] What is the range of the function $y = H(t)$ in the context of this problem? Give your answer using either interval notation or inequalities.

Solution: The values of $H(t)$ that are relevant in the context of this problem are the height of the package from the moment it is thrown from the airplane until it hits the ground, $0 \leq y \leq 160$.

d. [4 points] Another package is released from an airplane at a higher altitude. In this case, the downward velocity $V(t)$ (in meters per second) of the package t seconds after it was released is given by the function

$$V(t) = 50 - 50e^{-0.2\,t}$$

How long does it take for the package to have a downward velocity of 30 meters per second? Find your answer algebraically. Show all your work step by step. Your answer must be in **exact form**.

Solution:

$$\begin{aligned} 50 - 50e^{-0.2\,t} &= 30 \\ e^{-0.2\,t} &= \frac{2}{5} \\ -0.2\,t &= \ln\left(\frac{2}{5}\right) \\ t &= -5\ln\left(\frac{2}{5}\right) \end{aligned}$$