8. [9 points]

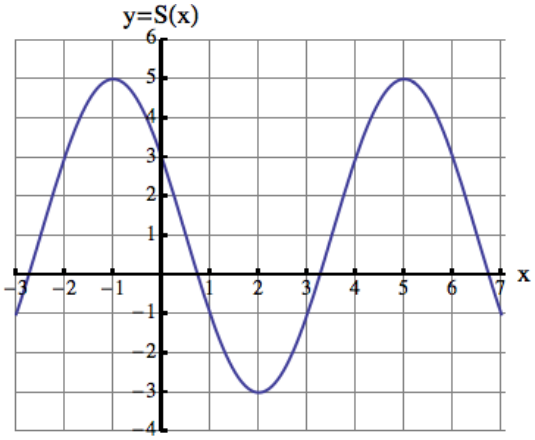
a. [4 points] A population of butterflies in a botanical garden has been found to oscillate sinusoidally. The population of butterflies reaches a maximum of 2000 butterflies followed by a minimum of 750 butterflies two months later. Let $B(t)$ be the amount of butterflies in the botanical garden at time t (in months). Find the amplitude, midline and period of the periodic function $y = B(t)$.

Amplitude: _____.
Period: _____.
Midline: _____.

Solution:
Amplitude$=\dfrac{2000-(750)}{2}=625.$ Midline: $y=\dfrac{2000+(750)}{2}=1375$

Period$=4$ (months).

b. [5 points] The graph of a sinusoidal function $y = S(x)$ is shown below. Find a formula for $S(x)$.



Solution: Amplitude$=\dfrac{5-(-3)}{2}=4.$ Midline: $y=\dfrac{5+(-3)}{2}=1$ Period$=6$.

Four possible solutions (among many others):

- $y=4\sin\left(\dfrac{2\pi}{6}(x+2.5)\right)+1=4\sin\left(\dfrac{\pi}{3}(x+2.5)\right)+1$
- $y=-4\sin\left(\dfrac{2\pi}{6}(x-0.5)\right)+1=-4\sin\left(\dfrac{\pi}{3}(x-0.5)\right)+1$
- $y=4\cos\left(\dfrac{2\pi}{6}(x+1)\right)+1=4\cos\left(\dfrac{\pi}{3}(x+1)\right)+1$
- $y=-4\cos\left(\dfrac{2\pi}{6}(x-2)\right)+1=-4\cos\left(\dfrac{\pi}{3}(x-2)\right)+1$