9. [13 points] Three scientists, Laura, Emily and Patrick studied the growth of certain cells using three different lab techniques for cell growth. Suppose that they started their experiments at the same time. Let $L(t)$, $E(t)$ and $P(t)$ be the number of cells t hours after noon using Laura's, Emily's and Patrick's techniques, respectively. All the questions should be solved algebraically step by step.

a. [2 points] The amount of cells $L(t)$ increased by 315 cells every two hours. Find a formula for $L(t)$ if there were 2150 cells in Laura's experiment at 3 pm.

Solution: $L(t) = 2150 + \frac{315}{2}(t-3) = \frac{315}{2}t + \frac{3355}{2} = 157.5t + 1677.5$

b. [5 points] Emily noticed that by applying her technique, the amount of cells doubled every 4 hours.

i) Find a formula for $E(t)$ in **exact form** if there were 1500 cells at 3 pm.

Solution: $E(t) = ab^t$, then

$$
\begin{aligned}
2a &= ab^4 \\
2 &= b^4 \qquad b = 2^{\frac{1}{4}} \\
1500 &= a2^{\frac{3}{4}} \\
a &= \frac{1500}{2^{\frac{3}{4}}} = 1500(2^{-\frac{3}{4}}). \qquad E(t) = 1500(2^{-\frac{3}{4}})2^{\frac{1}{4}t}
\end{aligned}
$$

ii) What is the continuous hourly growth rate of $E(t)$? Give your answer in exact form or accurate to at least three decimal places. Show all your work.

Solution: $b = e^k$, then $k = \ln b$, hence $k = \ln(2^{\frac{1}{4}}) \approx 0.173$. $k = 0.173$ or 17.3%.

c. [5 points] Patrick notices that the amount of cells in his experiment $P(t)$ is a power function. Find a formula for $P(t)$ if there are 2000 cells at 3 pm and 3000 at 5 pm. Show all your work step by step.

$P(t) =$ ______________________.

Solution: Since $P(t) = kt^p$, then

$$2000 = k3^p \qquad 3000 = k5^p$$

$$\frac{3000}{2000} = \frac{5^p}{3^p} = \left(\frac{5}{3}\right)^p$$

$$\frac{3}{2} = \left(\frac{5}{3}\right)^p \qquad \ln\left(\frac{3}{2}\right) = p\ln\left(\frac{5}{3}\right) \qquad p = \frac{\ln\left(\frac{3}{2}\right)}{\ln\left(\frac{5}{3}\right)} \approx 0.793.$$

$$2000 = k3^{0.793} \qquad k = \frac{2000}{3^{0.793}} \approx 836.898.$$

$$P(t) = 836.898t^{0.793}.$$

d. [1 point] Which of the three increasing functions $L(t)$, $E(t)$ and $P(t)$ grows fastest as $t \to \infty$? Circle your answer. You do not need to justify your answer.

$L(t)$ $\qquad$ $\mathbf{E(t)}$ $\qquad$ $P(t)$ $\qquad$ Not enough information to conclude.

Solution: $E(t)$: Increasing exponential functions increase faster than linear and power functions.