

3. [11 points]

- a. [4 points] A store sells bananas. For an order of less than 30 pounds of bananas, the store charges 45 cents per pound of bananas. On a purchase of 30 pounds or more, the store charges 34 cents per pound of bananas plus an additional fee of 3.30 dollars for packaging costs. A minimum of one pound of bananas is required on every purchase. Let $C(x)$ be the cost (in **dollars**) of buying x pounds of bananas. Find a formula for $C(x)$ as a piecewise-defined function.

Solution:

$$C(x) = \begin{cases} 0.45x & \text{for } 1 \leq x < 30 \\ 3.30 + 0.34x & \text{for } 30 \leq x \end{cases}$$

- b. [7 points] There is a fire in the forest. The amount of forest burnt (in km^2) increases exponentially at a continuous growth rate of 40 percent per hour. Authorities estimate that 12 km^2 of forest were burnt half an hour after the fire started. Let $B(t)$ be the total area burnt by the fire (in km^2) t hours after the fire started.

i) Find a formula formula for $B(t)$. Your formula has to be written in **exact form**.

Solution: The function $B(t) = ae^{0.4t}$ satisfies $B(0.5) = 12$, then

$$\begin{aligned} 12 &= ae^{0.2} \\ a &= \frac{12}{e^{0.2}} \end{aligned}$$

Then $B(t) = \frac{12}{e^{0.2}}e^{0.4t} = 12e^{0.4t-0.2}$.

ii) What is the hourly percent rate at which the fire burns the forest? Your answer must be written in **exact form** or accurate up to the first three decimals.

Solution: Since $b = e^{0.4} = 1 + r$, then $r = e^{0.4} - 1 \approx .492$ (or $100(e^{0.4} - 1) \approx 49.182$ percent).

iii) What is the doubling time of the function $B(t)$? Your answer must be found algebraically and written in **exact form**. Show all your work.

Solution:

$$\begin{aligned} 2a &= ae^{0.4t} \\ e^{0.4t} &= 2 \\ t &= \frac{1}{0.4} \ln 2 \text{ hours} \end{aligned}$$