3. [11 points]

a. [4 points] A store sells bananas. For an order of less than 30 pounds of bananas, the store charges 45 cents per pound of bananas. On a purchase of 30 pounds or more, the store charges 34 cents per pound of bananas plus an additional fee of 3.30 dollars for packaging costs. A minimum of one pound of bananas is required on every purchase. Let \( C(x) \) be the cost (in dollars) of buying \( x \) pounds of bananas. Find a formula for \( C(x) \) as a piecewise-defined function.

Solution:

\[
C(x) = \begin{cases} 
0.45x & \text{for } 1 \leq x < 30 \\
3.30 + 0.34x & \text{for } 30 \leq x 
\end{cases}
\]

b. [7 points] There is a fire in the forest. The amount of forest burnt (in km\(^2\)) increases exponentially at a continuous growth rate of 40 percent per hour. Authorities estimate that 12 km\(^2\) of forest were burnt half an hour after the fire started. Let \( B(t) \) be the total area burnt by the fire (in km\(^2\)) \( t \) hours after the fire started.

i) Find a formula for \( B(t) \). Your formula has to be written in exact form.

Solution: The function \( B(t) = ae^{0.4t} \) satisfies \( B(0.5) = 12 \), then

\[
12 = ae^{0.2} \quad a = \frac{12}{e^{0.2}}
\]

Then \( B(t) = \frac{12}{e^{0.2}}e^{0.4t} = 12e^{0.4t-0.2} \).

ii) What is the hourly percent rate at which the fire burns the forest? Your answer must be written in exact form or accurate up to the first three decimals.

Solution: Since \( b = e^{0.4} = 1 + r \), then \( r = e^{0.4} - 1 \approx .492 \) (or \( 100(e^{0.4} - 1) \approx 49.182 \) percent).

iii) What is the doubling time of the function \( B(t) \)? Your answer must be found algebraically and written in exact form. Show all your work.

Solution:

\[
2a = ae^{0.4t} \\
e^{0.4t} = 2 \\
t = \frac{1}{0.4} \ln 2 \text{ hours}
\]