4. [14 points] The noise level of the sound of a fire alarm at a factory oscillates between a maximum of 120 decibels to a minimum of 70 decibels. It takes the alarm 10 seconds to go from its maximum to its minimum noise level. Let $y = f(t)$ be the noise level of the sound of the alarm (in decibels) t seconds after it is activated. Suppose that $f(t)$ is a **sinusoidal** function and that $f(0) = 70$.

a. [6 points] Find the period, the amplitude, the midline and formula of the function $y = f(t)$. Include units.

Solution: Period= 20 seconds Amplitude= $\frac{120-70}{2} = 25$ decibels

Midline: $y = \frac{120+70}{2} = 95$

$f(t) = 95 - 25\cos\left(\frac{\pi}{10}t\right)$

b. [2 points] The factory considered the alarm to be too loud. They modified their alarm by reducing its noise level by 20 percent and they doubled the length of time it takes for the alarm to go from its lowest to its maximum noise level. Let $g(t)$ be the noise level (in decibels) of the modified alarm t seconds after it is activated. Find a formula for $g(t)$ in terms of the function $f(t)$ or as a sinusoidal function in terms of t.

Solution: $g(t) = 0.8f\left(\frac{1}{2}t\right)$.

c. [6 points] The noise level $F(t)$ of the sound (in decibels) of a fire alarm at a different factory t seconds after it is activated is given by

$$F(t) = 80 + 35\sin\left(\frac{\pi}{28}t\right)$$

It is known that sounds with intensity above 110 decibels are dangerous for the human ear. For which values of $0 \le t \le 70$ will the intensity of the fire alarm be exactly 110 decibels? *You must find your answer(s) algebraically and write them in* ***exact form****. Show all your work.*

Solution:

$$110 = 80 + 35\sin\left(\frac{\pi}{28}t\right)$$

$$\sin\left(\frac{\pi}{28}t\right) = \frac{30}{35}$$

$$t = \frac{28}{\pi}\sin^{-1}\left(\frac{30}{35}\right)$$

$$t_1 = \frac{28}{\pi}\sin^{-1}\left(\frac{30}{35}\right) \qquad t_2 = 28 - t_1 \qquad t_3 = t_1 + 56.$$