6. [13 points]

a. [6 points] Let $P(x) = \dfrac{(x^2-2x-4)^2}{7x^4-1400}$.

Find the following quantities. Your answers must be found algebraically and written in **exact form**. Show your work.

i) Find the equation(s) of the vertical asymptotes of $P(x)$:

Solution: Vertical asymptotes when $7x^4 - 1400 = 0$ or $x = \pm\sqrt[4]{200}$.

ii) Find the horizontal intercepts of $P(x)$.

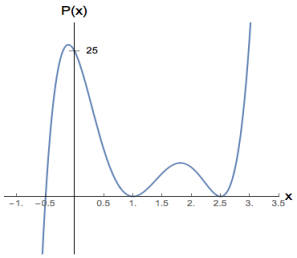
Solution: Horizontal intercepts when $x^2 - 2x - 4 = 0$. Using the quadratic formula

$$x = \frac{2 \pm \sqrt{4-4(-4)}}{2} = 1 \pm \sqrt{5}.$$

iii) Does the graph of $y = P(x)$ have horizontal asymptotes? If so, write its equation, otherwise write "None".

Solution: Since the function $P(x)$ behaves as $x \to \pm\infty$ similarly to $\frac{x^2)^2}{7x^4} = \frac{1}{7}$, then the graph of $P(x)$ has a horizontal asymptote at $y = \dfrac{1}{7}$.

b. [5 points] Find the formula for the polynomial $P(x)$ of degree five shown below



Solution: Looking at the graph $P(x) = a(x+0.5)(x-1)^2(x-2.5)^2$. Since $P(0) = 25$, then $25 = a(0.5)(-1)^2(-2.5)^2$. This yields $a = 8$.

Hence $P(x) = 8(x+0.5)(x-1)^2(x-2.5)^2$

c. [2 points] Write the equation of a rational function $R(x)$ that satisfies both conditions below:

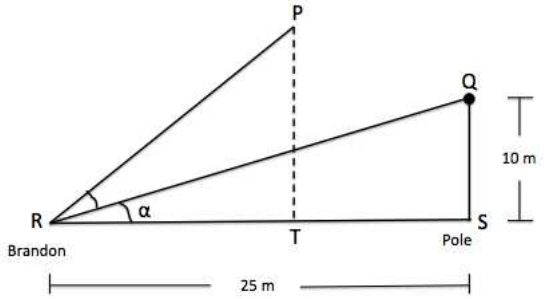
i) The graph of $y = R(x)$ has a vertical asymptote at $x = 1$.

ii) $\lim_{x\to\infty} R(x) = \infty$

There may be more than one correct answer. You only need to find one of them.

Solution: The rational function $R(x) = \dfrac{p(x)}{q(x)}$ has to be undefined at $x = 1$ and the degree of $P(x)$ has to be larger than the degree of $q(x)$. One example of a rational function satisfying both conditions is $R(x) = \dfrac{x^2}{x-1}$.

7. [5 points] Brandon takes a picture of a bird standing (at point Q) on top of a 10 meter high pole. The pole (at point S) is 25 meters away from where Brandon stands (at point R).



i) Find the value of the angle α (the angle SRQ) measured in **radians**. *Your answer must be written in* ***exact form*** *or accurate up to two decimals. Show all your work.*

Solution: Since $\tan(\alpha) = \frac{10}{25}$ then $\alpha = \tan^{-1}\left(\dfrac{10}{25}\right) \approx 0.3805$.

ii) The bird flies to the top of a nearby tree (point P). Brandon takes a second picture of the bird once it is at the top of the tree. The length of the segment RP is 20 meters. What is the height of the tree, the length of the segment PT, if you know that the angle QRP measures 0.75 radians.
Your answer must be written in ***exact form*** *or accurate up to two decimals. Show all your work.*

Solution: In this case $\sin(\alpha + 0.75) = \dfrac{PT}{20}$, then $PT = 20\sin(\alpha + 0.75) \approx 18.092$