3. [9 points]

a. [4 points] A residential community started a paper recycling program in 2002. According to their records, the community recycled 4000 and 12000 lbs of paper in 2005 and 2013, respectively. Let $P(t)$ be the amount of paper recycled by this community (in lbs) t years after 2002. Find a formula for $P(t)$ if you assume that it is a power function. Your answer must be written in **exact** form.

Solution: Since $P(t) = kt^p$ then

$$k(3^p) = 4000 \qquad k(11^p) = 12000.$$

$$\left(\frac{11}{3}\right)^p = 3.$$

$$p \ln\left(\frac{11}{3}\right) = \ln(3) \qquad p = \frac{\ln(3)}{\ln\left(\frac{11}{3}\right)}$$

$$k = \frac{4000}{3^{\frac{\ln(3)}{\ln\left(\frac{11}{3}\right)}}} = (4000)3^{-\frac{\ln(3)}{\ln\left(\frac{11}{3}\right)}}.$$

Hence $P(t) = (4000)3^{-\frac{\ln(3)}{\ln\left(\frac{11}{3}\right)}} t^{\frac{\ln(3)}{\ln\left(\frac{11}{3}\right)}}$.

b. [5 points] Let $W(t)$ be the water consumption of the residential community, in millions of gallons, t years after 2005. The table below shows some values of $W(t)$

t	2	5	8
$W(t)$	5.38	10.51	20.52

Note: The values in the table have been rounded to the nearest 0.01.

Assume that the function $W(t)$ increases exponentially. Your answers should be written in exact form or round your answers to the nearest 0.01.

i) What is the annual percent rate of the function $W(t)$? Show all your work.

Solution: Since $W(t) = ab^t$, then $\frac{W(5)}{W(2)} = \frac{ab^5}{ab^2} = b^3 = \frac{10.51}{5.38}$.

Then $b = \sqrt[3]{\frac{10.51}{5.38}} \approx 1.25$ and $r = b - 1 = \sqrt[3]{\frac{10.51}{5.38}} - 1 \approx 0.25$.

ii) What is the annual continuous rate of $W(t)$?

Solution: $k = \ln(b) = \ln\left(\sqrt[3]{\frac{10.51}{5.38}}\right) \approx 0.22$.