6. [12 points]

a. [2 points] Let \( f(x) \) be an odd function whose domain is all real numbers except \( x = 3 \) and \( x = -3 \). Suppose that \( \lim_{{x \to 3^+}} f(x) = \infty \) and \( \lim_{{x \to -\infty}} f(x) = -3 \). Compute the following limits. Write “NI” if not enough information has been provided to answer the question.

\[
\begin{align*}
\text{Solution:} & \quad \lim_{{x \to -\infty}} f(x) = 3 \\
& \quad \lim_{{x \to -3^-}} f(x) = -\infty
\end{align*}
\]

b. [2 points] Which of the following functions dominates the other functions as \( x \to \infty \)? Circle your answer.

\[
\begin{align*}
\text{Solution:} & \quad y = 20x^{500} \\
& \quad y = 4(1.05)^x \\
& \quad y = 1000 \log(x) \\
& \quad y = 2e^{0.05x}.
\end{align*}
\]

c. [2 points] Fill in the blank space. Your answer may depend on the constant \( B \).

\[
\begin{align*}
\text{If } B \text{ is a constant, then } & \quad \frac{3^x + Bx^2}{4x^2 + Bx + 10^x} \to \frac{B}{4} \quad \text{as } x \to -\infty.
\end{align*}
\]

\[
\text{Solution:}
\]

d. [6 points] Consider the function \( y = h(x) = 2 + 3 \log(4x + 10) \) with domain \( x \geq 0 \).

i) What is the range of \( h(x) \) given that its domain is \( x \geq 0 \)? Your answer must be written using interval notation or inequalities.

\[
\text{Solution:} \quad \text{Range of } h(x): [5, \infty)
\]

ii) Find a formula for \( h^{-1}(y) \).

\[
\text{Solution:} \\
\begin{align*}
y & = 2 + 3 \log(4x + 10) \\
y - 2 & = 3 \log(4x + 10) \\
\frac{y - 2}{3} & = \log(4x + 10) \\
4x + 10 & = 10^{\frac{y-2}{3}} \\
4x & = 10^{\frac{y-2}{3}} - 10 \\
x & = \frac{10^{\frac{y-2}{3}} - 10}{4} = h^{-1}(y).
\end{align*}
\]