6. [12 points]

a. [2 points] Let $f(x)$ be an odd function whose domain is all real numbers except $x = 3$ and $x = -3$. Suppose that $\lim_{x\to 3^+} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = -3$. Compute the following limits. Write "NI" if not enough information has been provided to answer the question.

Solution: $\lim_{x\to-\infty} f(x) = 3$ $\lim_{x\to -3^-} f(x) = -\infty$

b. [2 points] Which of the following functions dominates the other functions as $x \to \infty$? Circle your answer.

Solution: $y = 20x^{500}$ $y = 4(1.05)^x$ $y = 1000\log(x)$ $\boxed{y = 2e^{0.05x}.}$

c. [2 points] Fill in the blank space. Your answer may depend on the constant B.

If B is a constant, then $\dfrac{3^x + Bx^2}{4x^2 + Bx + 10^x} \longrightarrow \dfrac{B}{4}$ as $x \longrightarrow -\infty$.

Solution:

d. [6 points] Consider the function $y = h(x) = 2 + 3\log(4x + 10)$ with domain $x \geq 0$.

i) What is the range of $h(x)$ given that its domain is $x \geq 0$? Your answer must be written using interval notation or inequalities.

Solution: Range of $h(x)$: $[5, \infty)$

ii) Find a formula for $h^{-1}(y)$.

Solution:

$$\begin{aligned} y &= 2 + 3\log(4x + 10) \\ y - 2 &= 3\log(4x + 10) \\ \frac{y-2}{3} &= \log(4x + 10) \\ 4x + 10 &= 10^{\frac{y-2}{3}} \\ 4x &= 10^{\frac{y-2}{3}} - 10 \\ x &= \frac{10^{\frac{y-2}{3}} - 10}{4} = h^{-1}(y). \end{aligned}$$